Circular economy innovations in a 2-area input-output stock-flow consistent dynamic model

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– First draft –

Abstract

The aim of this paper is twofold. First, an ecological 2-area input-output stock-flow consistent dynamic model is developed from scratch, in which money is endogenously created, prices are defined in a Sraffa-like fashion, and the economy is split into different industries. Second, the model is used to test both domestic and cross-area impacts of "circular economy" innovations on the economy, the society and the ecosystem.

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Contents

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1 Introduction

While dynamic stochastic general equilibrium (DSGE) models have dominated macroeconomics since the mid-1990s, seldom have these models been used to deal with environmental topics. This is no surprise, as the small size of DSGE models and their "harmonic oscillator"-like dynamics make them unfit for analyzing climate change-related issues – let alone biodiversity loss, rebound effects, lock-in effects, commons management, circular economy policies, etc. Standard neoclassical growth models (NG) are also unsuitable, as environmental variables do not affect the structural determinants of economic growth (Rivera et al. 2018[6]). As a result, two additional classes of models are usually employed by mainstream economists to address environmental questions: *a*) integrated assessment models (IAM); and *b*) computational general equilibrium models (CGE).

IAMs are specifically designed to integrate the economy with the biosphere and atmosphere. They are usually made up of a number of social, economic, and environmental "modules", which allow quantifying climate-change mitigation scenarios (process-based IAMs) or calculating the social cost of carbon (cost-benefit IAMs). Popular though they are, most IAMs (particularly, cost-benefit IAMs) share the same controversial presuppositions of DSGE and NG models: there is a unique and stable socially-optimal equilibrium in the long run; and the process of adjustment is driven by the decisions made by a hyper-rational representative individual agent who maximizes an intertemporal utility function subject to economic, technical, and/or environmental constraints. Turning to standard CGE models, these are large scale models whose accounting structure is based on input-output tables and the national accounts. As such, they are well-equipped to factor in a variety of social, economic, and environmental variables. Unfortunately, CGE behavioral equations are also based on neoclassical general equilibrium principles.[1] Besides, these models can only compare the economy before and after a shock, thus neglecting the transition from one state to the other.

Is there an alternative? The answer is yes, there is. In fact, there are two: *a*) non-neoclassical or demand-driven input-output models (IO); and *a*) stock-flow consistent macroeconomic models (SFC). IO and SFC models are convenient tools to address environmental issues, as they do not imply any unreasonable behavioral assumptions or any *a priori* equilibrium dynamics. More precisely, IO models shed light on interdependencies across industries and, like standard CGE models, can be easily extended to include ecosystem-related variables. Their main limitation is that are not strictly-defined "dynamic models", as they only compare two different states of the world, thus ignoring the transition between them. SFC models can be regarded as a specific class of system dynamics tools, mostly developed by post-Keynesian macroeconomists since the early 2000s. SFC models have gained momentum in ecological macroeconomics in the last decade, because they allow for a consistent and comprehensive integration of the flows and the stocks of the economy and the ecosystem (e.g. Carnevali et al. 2019[3]). This feature makes them one of the most flexible and versatile tools to simulate, analyse, and compare alternative environmental policy scenarios. Their main limitation is that they only consider aggregate output, so neglecting the interdependencies between different industries.

2 The formal model

Each economic area is made up of five macroeconomic sectors: *a*) households (which are then split into wage-earners and rentiers); *b*) private production firms; *c*) the government sector; *d*) commercial banks; and *e*) the central bank. Both trade and financial flows takes place across the two areas.

Households are the final recipients of both labour incomes (wages) and capital incomes

[1] Some CGE models depart from the standard assumptions of flexible prices and full employment by introducing market frictions. However, their core dynamics is still that of a general equilibrium model.

(profits and interest payments). They buy consumption goods based on both their disposable income and their stock of net wealth. Household savings are made up of cash (domestic currency), bank deposits, domestic and foreign corporate shares, and domestic and foreign government bills. The purchase of durable goods is partially funded through personal loans.

There are four traditional industries (manufacturing, agriculture, services, and waste management, respectively) under the baseline scenario, in which firms produce three outputs (and waste) by means of the same products used as inputs.

For the sake of simplicity, real supplies always adjust to real demands. As a result, firms hold no inventories. However, firms accumulate fixed capital and finance their production plans using bank loans. As mentioned, corporate incomes are entirely distributed to households. Bank deposits are created as long as banks grant loans to the private sector and/or on demand. Cash is issued by the central bank as the government sector runs into budget deficits and/or commercial banks obtain advances.

2.1 Households

If we use the superscript z to define each area and f to define the other area (that is, the foreign sector), households' domestic consumption in real terms is:

$$c^z = \alpha_1^z \cdot \frac{YD_w^z}{E(p_A^z)} + \alpha_2^z \cdot \frac{YD_c^z}{E(p_A^z)} + \alpha_3^z \cdot \frac{V_{-1}^z}{p_{A,-1}^z} \tag{1}$$

where p_A^z is a consumer price index, while α_1^z, α_2^z and α_3^z are the propensities to consume out of disposable labour income (YD_w^z), disposable capital income (YD_c^z) and net wealth (V^z), respectively.[2]

Disposable income is net domestic incomes from firms and banks *plus* received interests on bank deposits and government debt *plus* capital gains on holdings of foreign bills and shares *minus* taxes and interest payments on personal loans:

$$\begin{aligned} YD^z = {} & WB^z + DIV^z + FB^z + \\ & + r_{m,-1}^z \cdot M_{h,-1}^z + r_{b,-1}^z \cdot B_{s,z,-1}^z + xr_{-1}^f \cdot r_{b,-1}^f \cdot B_{s,z,-1}^f + \\ & + \Delta xr^f \cdot (B_{s,z,-1}^f + E_{s,z,-1}^f) + \\ & - r_{h,-1}^z \cdot L_{h,-1}^z - T^z \end{aligned} \tag{2}$$

where WB^z is the wage bill, DIV^z is distributed profits of firms, F^z is bank profits (which are assumed to be fully distributed), r_m^z is the interest rate paid on bank deposits (M_h^z), r_b^z is the interest rate on domestic government bills held by domestic households ($B_{s,z}^z$), xr^f is the nominal exchange rate,[3] r_b^f is the interest rate on foreign government bills held by domestic households ($B_{s,z}^f$), $E_{s,z}^f$ is domestic holdings of foreign shares, r_h^z is the interest rate on personal loans granted to domestic households (L_h^z), and T^z is income tax payments.

More precisely, disposable labour income in each area is:

$$YD_w^z = WB^z \cdot (1 - \theta_w^z) \tag{3}$$

where θ_w^z is the average tax rate on labour incomes.

Total disposable capital income is:

$$YD_c^z = YD^z - YD_w^z \tag{4}$$

[2] Purely adaptive price expectations are assumed in the baseline scenario, so that: $E(p_A^z) = p_{A,-1}^z$. Besides, the impact of the so-called "inflation tax" on real disposable income is ignored.

[3] Exchange rates are quoted indirectly. As a result, xr^z is the price of one unit of domestic currency expressed in foreign currency, whereas, for the 'home' area, xr^f is the price of one unit of foreign currency expressed in domestic currency.

Net private wealth accumulated in each area is:

$$V^z = V^z_{-1} + YD^z - c^z \cdot p^z_A \tag{5}$$

The stock of wealth increases as households save. Portfolio decisions (that is, the way in which net wealth is held) are discussed in the subsection 2.7. Consumption composition is discussed below (subsection 2.2).

2.2 Production firms (current)

The final demand faced by production firms is made up of household consumption, corporate investment in fixed capital, government spending, and net export. Considering 10 industries and products at the global level, the demand for final goods and services in each area is:

$$\begin{aligned}\mathbf{d}^z &= \beta^z \cdot c^z + \iota^z \cdot i^z_d + \sigma^z \cdot gov^z + \eta^f_z \cdot exp^z + \eta^z \cdot imp^z = \\ &= \begin{pmatrix} d^z_1 \\ \vdots \\ d^z_{10} \end{pmatrix} = \begin{pmatrix} \beta^z_1 \\ \vdots \\ \beta^z_{10} \end{pmatrix} \cdot c^z + \begin{pmatrix} \iota^z_1 \\ \vdots \\ \iota^z_{10} \end{pmatrix} \cdot i^z_d + \begin{pmatrix} \sigma^z_1 \\ \vdots \\ \sigma^z_{10} \end{pmatrix} \cdot gov^z + \begin{pmatrix} \eta^f_{1,z} \\ \vdots \\ \eta^f_{10,z} \end{pmatrix} \cdot exp^z + \\ &\qquad - \begin{pmatrix} \eta^z_1 \\ \vdots \\ \eta^z_{10} \end{pmatrix} \cdot imp^z\end{aligned} \tag{6}$$

where i^z_d is real corporate demand for investment, gov^z is real government consumption, exp^z is real gross export of final goods, imp^z is real gross import of final goods, β^z is the vector of household consumption shares (with: $\sum_{s=1}^{10} \beta^z_s = 1$), ι^z is the vector of investment shares (with: $\sum_{s=1}^{10} \iota^z_s = 1$), σ^z is the vector of government spending shares (with: $\sum_{s=1}^{10} \sigma^z_s = 1$), η^f_z is the vector of export shares (with: $\sum_{s=1}^{10} \eta^f_{z,s} = 1$),[4] and η^z is the vector of import shares (with: $\sum_{s=1}^{10} \eta^z_s = 1$).

Note that it is assumed that there is only a direct demand for manufacturing goods, agricultural goods and services. As a result, considering 5 domestic industries per area implies that the demand vectors of the two areas will look like:

$$\mathbf{d}^z = \begin{pmatrix} d^z_1 > 0 \\ d^z_2 > 0 \\ d^z_3 > 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \quad \mathbf{d}^f = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ d^f_1 > 0 \\ d^f_2 > 0 \\ d^f_3 > 0 \\ 0 \\ 0 \end{pmatrix}$$

Unlike other spending shares, the composition of household consumption is endogenous. More precisely, the share of services to total consumption is assumed to increase as disposable income (expressed in real terms, using the price of services) increases, whereas the share of manufacturing goods remains constant. Using subscript 1 for domestic manufacturing, 2 for domestic agriculture, and 3 for domestic services, real domestic consumption shares are:

$$\beta^z_1 = \bar{\beta}^z_1 \tag{7}$$

$$\beta^z_2 = 1 - \beta^z_1 - \beta^z_3 \tag{8}$$

[4] For each area, the vector of export shares mirror the vector of import shares of the other area.

$$\beta_3^z = \beta_{3,-1}^z + \beta_{31}^z \cdot \frac{YD_{w,-1}^z}{p_{3,-1}^z} + \beta_{32}^z \cdot \frac{YD_{c,-1}^z}{p_{3,-1}^z} \tag{9}$$

where β_{31}^z and β_{32}^z are positive coefficients.

Once final demands are known, the gross output vector can be defined as:

$$\mathbf{x}^z = \begin{pmatrix} x_1^z \\ \vdots \\ x_{10}^z \end{pmatrix} = \mathbf{A} \cdot \mathbf{x}^z + \mathbf{d}^z$$

from which:

$$\mathbf{x}^z = (\mathbf{I} - \mathbf{A})^{-1} \cdot \mathbf{d}^z \tag{10}$$

where $\mathbf{I}$ is the identity matrix and $\mathbf{A}$ is the global matrix of technical coefficients, defined as:

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{110} \\ a_{21} & a_{22} & \cdots & a_{210} \\ \cdots & \cdots & \cdots & \cdots \\ a_{101} & a_{102} & \cdots & a_{1010} \end{pmatrix}$$

As usual, a_{ij} (with $i, j = 1, 2, ..., 10$) is the quantity of product i necessary to produce one unit of product j. Therefore, each column j of $\mathbf{A}$ is associated with an industry, a the technique of production, and a product.[5] More precisely, columns 1 to 5 are associated with industries of the first area, whereas columns 6 to 10 are associated with industries of the second area. Similarly, rows 1 to 5 shows outputs produced by industries of the first area used as inputs by other industries, whereas rows 6 to 10 shows outputs produced by industries of the second area used as inputs by other industries. We refer to Table 3 for an example.

The monetary value of gross domestic output is the product of the unit price vector and the output vector:

$$Y^z = \mathbf{p}^{zT} \cdot \mathbf{x}^z \tag{11}$$

where $\mathbf{p}^z$ is the price vector and the superscript "T" stands for the transpose of the matrix (hence $\mathbf{p}^{zT}$ is a row vector).

The net income or value added for each domestic economy matches aggregate nominal demand for final products and services, net of VAT and tariffs:

$$YN^z = c^z \cdot p_A^z + i_d^z \cdot p_I^z + gov^z \cdot p_G^z + EXP^z - IMP^z - VAT^z - TAR^z \tag{12}$$

where p_I^z is an investment price index, p_G^z is a government spending price index,[6] EXP^z is nominal export of final goods, IMP^z is nominal import of final goods, VAT^z is VAT revenues, and TAR^z is tariff revenues.

Total corporate profit in each area is:

$$FF^z = YN^z - WB^z - r_{l,-1}^z \cdot L_{F,-1}^z - AF^z \tag{13}$$

[5] Notice that the term $(\mathbf{I} - \mathbf{A})^{-1}$ is a matrix too. It is named the *Leontief inverse* and shows the multipliers, that is, the successive changes in production processes triggered by an initial change in final demands. As is well known, the Leontief inverse matrix can be expressed as a sum of power series (Waugh 1950[8]), that is: $(\mathbf{I} - \mathbf{A})^{-1} = \mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \mathbf{A}^3 + \ ... \ + \mathbf{A}^t + \ ... \ = \sum_{t=0}^{\infty} \mathbf{A}^t$.

[6] As we are explaining in subsection 2.8, p_I^z is the average price of investment goods and p_G^z is the average price of goods purchased by the government sector.

where r_l^z is the interest rate on loans obtained by production firms (L_F^z), and AF^z are amortization funds.

Productions firms can retain a supplementary share of profits, in addition to using funds for amortization:

$$FF_u^z = \omega^z \cdot FF^z \tag{14}$$

where ω^z is the percentage of (additional) undistributed profits of firms.

2.3 Production firms (capital)

Firms need fixed capital (in addition to labour and circulating capital inputs) to produce. It is assumed that each industry has its own capital requirement. The target stock of fixed capital, expressed in real terms, is therefore:

$$k^{z*} = \mathbf{p}_{-1}^{zT} \cdot \left(\mathbf{h}^z \odot \mathbf{x}_{-1}^z\right) \cdot \frac{1}{p_{I,-1}^z} \tag{15}$$

where $\mathbf{h}^z = \{h_j^z\}$ is the column vector of industry-specific target capital to output ratios.[7]

The real gross investment adjusts in such a way to bridge the gap between the actual capital stock (at the beginning of the period) and its target level:

$$i_d^z = \gamma^z \cdot (k^{z*} - k_{-1}^z) + da^z \tag{16}$$

where γ^z defines the speed of adjustment, and da^z is real capital depreciation.

The current capital stock depreciates according to a constant ratio, δ^z, so that:

$$da^z = \delta^z \cdot k_{-1}^z \tag{17}$$

It follows that the real stock of current fixed capital in each area is:

$$k^z = k_{-1}^z + i_d^z - da^z \tag{18}$$

Amortization funds are used to fund the replacement of depleted capital:

$$AF^z = da^z \cdot p_I^z - k^z \cdot \Delta p_I^z \tag{19}$$

The stock of bank loans obtained by production firms is defined as a residual variable:

$$L_F^z = L_{F,-1}^z + i_d^z \cdot p_I^z - AF^z - FF_u^z - \Delta E_s^z \tag{20}$$

where E_s^z is the nominal value of the stock of shares issued by production firms.

For the sake of simplicity, we assume that share issues are completely demand driven:

$$E_s^z = E_{h,z}^z + xr_f \cdot E_{h,f}^z \tag{21}$$

where $E_{h,z}^z$ is nominal stock of domestic shares held by domestic investors and $E_{h,f}^z$ is the portion held by foreign investors.

The supply of domestic shares to foreign investors, expressed in domestic currency, is therefore:

$$E_{s,f}^z = xr_f \cdot E_{h,f}^z \tag{22}$$

[7] Notice that k^* cannot be expressed in physical units. Rather, it is calculated by dividing the nominal stock of capital by the average price of investment goods. See subsection 2.8.

The return rate (in addition to percentage capital gains) on shares issued by production firms of each area is:

$$r_e^z = \frac{(1-\omega^z) \cdot FF^z}{E_s^z} \tag{23}$$

Finally, total dividends (from non-financial firms) received by investors in each area are:

$$DIV^z = (1-\omega^z) \cdot FF^z \cdot \frac{E_{h,z}^z}{E_s^z} + (1-\omega^f) \cdot FF^f \cdot \frac{E_{h,z}^f}{E_s^f} \tag{24}$$

2.4 Commercial banks

For the sake of simplicity, it is assumed that commercial banks are always ready to finance firms' production plans and to fund private investment and consumption expenditures. Supplied loans are, therefore, demand driven:

$$L_s^z = L_F^z + L_h^z \tag{25}$$

Banks provide deposits on demand:

$$M_s^z = M_h^z \tag{26}$$

Because of cash (or state money), deposits collected by the banks may exceed those created by granting loans to the firms. If this happens, banks hold government bills as the asset counterpart of extra-deposits. Conversely, if loans exceed deposits, banks request (and obtain) advances from the central bank:

$$if\ M_s^z \geq L_s^z\ then\ B_b^z = M_s^z - L_s^z\ and\ A_d^z = 0 \tag{27}$$

$$if\ M_s^z < L_s^z\ then\ B_b^z = 0\ and\ A_d^z = L_s^z - M_s^z \tag{28}$$

where A_d^z are advances obtained by commercial banks from the central bank.

It is assumed that the interest rate on advances is nil, banks have no costs of production, and there are no compulsory reserves. As a result, bank profits equal the difference between perceived interests on loans and bills and interest payments on deposits:

$$F_b^z = r_{l,-1}^z \cdot L_{F,-1}^z + r_{h,-1}^z \cdot L_{h,-1}^z + r_{b,-1}^z \cdot B_{b,-1}^z - r_{m,-1}^z \cdot M_{s,-1}^z \tag{29}$$

Unlike corporate profits, bank profits are entirely distributed to the households.

2.5 Government and central bank

Real government spending grows according to an exogenous rate:[8]

$$gov^z = gov_{-1}^z \cdot (1 + g_g^z) + gov_0^z \tag{30}$$

where g_g^z is the growth rate of government spending and gov_0^z is a shock component.

Income taxes collected by the government can be calculated using the average tax rate on households' labour and non-labour incomes. The corresponding revenue is therefore:

$$T^z = \theta_w^z \cdot WB^z + \theta_c^z \cdot (DIV^z + r_{m,-1}^z \cdot M_{h,-1}^z + r_{b,-1}^z \cdot B_{s,z,-1}^z + xr_{-1}^f \cdot r_{b,-1}^f \cdot B_{s,z,-1}^f) \tag{31}$$

where θ_c^z is the average tax rate on capital incomes in each area.

Government revenues from VAT and tariffs are, respectively:

$$VAT^z = [\mathbf{p}^z \odot \tau_{vat}^z \oslash (I + \tau_{vat}^z)]^T \cdot (\beta^z \cdot c^z) \tag{32}$$

[8] However, it is assumed that $g_g^z = 0$ in the baseline scenario.

$$TAR^z = \left[xr^f \cdot \mathbf{p}^f \odot \tau^z_{tar} \oslash (I + \tau^z_{tar})\right]^T \cdot (\eta^z \cdot imp^z) \tag{33}$$

where τ^z_{vat} and τ^z_{tar} are the vectors defining product-specific VAT rates and percentage tariffs, respectively.[9]

The government budget deficit in each area is:

$$DEF^z_g = gov^z \cdot p^z_G + r^z_{b,-1} \cdot B^z_{s,-1} - F^z_{cb} - T^z - VAT^z - TAR^z \tag{34}$$

where F_{cb} is the profit made by the central bank (seigniorage income) on its holdings of (both domestic and foreign) government securities, which is subsequently returned to the government sector.

The government sector issues bills as it runs into deficits:

$$B^z_s = B^z_{s,-1} + DEF^z_g \tag{35}$$

Advances to commercial banks are provided on demand:

$$A^z_s = A^z_d \tag{36}$$

Similarly, the supply of cash adjusts to the demand for cash:

$$H^z_s = H^z_h \tag{37}$$

This is the overall amount of state money that remains in circulation at the end of each period.

The stock of bills supplied to domestic investors is:

$$B^z_{s,z} = B^z_{h,z} \tag{38}$$

whereas the stock of bills supplied to foreign investors is:

$$B^z_{s,f} = xr^f \cdot B^z_{h,f} \tag{39}$$

The profit made by the central bank is:

$$F^z_{cb} = r^z_{b,-1} \cdot B^z_{cb,z,-1} + xr^f \cdot r^f_{b,-1} \cdot B^f_{cb,s,z,-1} \tag{40}$$

where $B^f_{cb,s,z}$ is the amount of foreign government bills held by the domestic central bank, expressed in foreign currency.

Finally, interest rates on bank deposits, government bills, loans to firms, and personal loans, are simply defined using different mark-ups (μ^z_s) over the policy rate (r^{*z}) set by the central bank, that is:

$$r^z_m = r^{*z} + \mu^z_m \tag{41}$$

$$r^z_b = r^{*z} + \mu^z_b \tag{42}$$

$$r^z_l = r^{*z} + \mu^z_l \tag{43}$$

$$r^z_h = r^{*z} + \mu^z_h \tag{44}$$

We assume that $r_h \geq r_l \geq r_b \geq r_m$ in the baseline scenario.

[9] Note that $\odot$ and $\oslash$ are the Hadamard multiplication and division, respectively, also called element-wise multiplication and division of matrices.

2.6 Population and the labour market

The employment level is determined by firms' demand for labour in each production process. More precisely, the number of workers hired in each industry is:

$$N_j^z = \frac{x_j^z}{pr_j^z} \tag{45}$$

$\forall j = 1, 2, ..., 5$, where pr_j^z is the product per worker in the j-th industry.

Total employment in each area is:

$$N^z = \mathbf{x}^{zT} \cdot \left[\begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} \oslash \mathbf{pr}^z \right] = \mathbf{x}^{zT} \cdot \mathbf{l}^z = \sum N_j^z \tag{46}$$

where $\mathbf{pr}^z$ is the vector of industry-specific labour productivities and therefore $\mathbf{l}^z$ is the column vector of labour coefficients.

The wage bill paid in each industry is:

$$WB_j^z = n_j^z \cdot w_j^z \tag{47}$$

$\forall j = 1, 2, ..., 5$, where w_j^z is the average money wage rate paid to employees of industry j.

The total wage bill is:

$$WB^z = \mathbf{N}^{zT} \cdot \mathbf{w}^z = \sum WB_j^z \tag{48}$$

where $\mathbf{N}^z$ and $\mathbf{w}^z$ are the vectors of industry-specific employees and wage rates, respectively. The equation above defines the overall cost of labour faced by private firms in each area.

The available labour force in each area's industries depends on an exogenous growth rate and the net inflow of immigrants from the other area:

$$\mathbf{POP}^z = \mathbf{POP}_{-1}^z \odot (I + \mathbf{g}_{pop}^z) + \mathbf{IMM}^z - \mathbf{IMM}^f \tag{49}$$

where $\mathbf{IMM}^z$ and $\mathbf{IMM}^f$ are the vectors defining inflows and outflows of labour-force in each area's industries.

Industry-specific unemployment rates in each area are:

$$un_j^z = 1 - \frac{N_{j,-1}^z}{POP_{j,-1}^z} \tag{50}$$

We assume that immigration inflows depend on three factors: a) the size of the population of the other area; b) the unemployment rate of the other area; c) the wage differential between the two areas. In formal terms, we obtain:

$$\mathbf{IMM}^z = \gamma_{imm,0}^z \odot \mathbf{POP}_{-1}^f + \gamma_{imm,1}^z \odot \mathbf{un}_{-1}^f + \gamma_{imm,2}^z \odot (\mathbf{w}_{-1}^z - \mathbf{w}_{-1}^f) \tag{51}$$

where $\gamma_{imm,0}^z$, $\gamma_{imm,1}^z$ and $\gamma_{imm,2}^z$ are positive coefficients.

Finally, gender segregation is assumed to be dependent on the wage level. Since men tend to occupy high-salary jobs, the percentage of female employees (ρ_j^z) in each industry reduces as the wage rate increases:

$$\rho_j^z = \rho_{0j}^z - \rho_{1j}^z \cdot (w_j^z - w_{j,-1}^z) \tag{52}$$

where ρ_{0j}^z and ρ_{1j}^z are positive coefficients.

2.7 Portfolio choices

Domestic household holdings of domestic government bills are defined by a Tobinesque portfolio equation:

$$\frac{B_{h,z}^z}{V^z} = \lambda_{10} + \lambda_{11} \cdot r_{b,-1}^z - \lambda_{12} \cdot \left(r_{b,-1}^f + \frac{\Delta xr^f}{xr^f} \right) - \lambda_{13} \cdot r_{m,-1}^z - \lambda_{14} \cdot \frac{YD^z}{V^z} - \lambda_{15} \cdot r_{e,-1}^z + \\ - \lambda_{16} \cdot \left(r_{e,-1}^f + \frac{\Delta xr^f}{xr^f} \right) \tag{53}$$

In plain words, the share of domestic government bills to net wealth in domestic households' portfolio increases as the interest rate on domestic government bills increases (this effect is captured by coefficient λ_{11}), and reduces as interest and return rates (including percentage capital gains) on other financial assets increase (coefficients λ_{12}, λ_{13}, λ_{15}, and λ_{16}). Besides, it reduces as the liquidity preference of domestic investors increases (coefficient λ_{14}).

Similarly, domestic household holdings of foreign government bills, domestic shares, and foreign shares, are, respectively:

$$\frac{B_{h,z}^f}{V^z} = \lambda_{20} - \lambda_{21} \cdot r_{b,-1}^z + \lambda_{22} \cdot \left(r_{b,-1}^f + \frac{\Delta xr^f}{xr^f} \right) - \lambda_{23} \cdot r_{m,-1}^z - \lambda_{24} \cdot \frac{YD^z}{V^z} - \lambda_{25} \cdot r_{e,-1}^z + \\ - \lambda_{26} \cdot \left(r_{e,-1}^f + \frac{\Delta xr^f}{xr^f} \right) \tag{54}$$

$$\frac{E_{h,z}^z}{V^z} = \lambda_{30} - \lambda_{31} \cdot r_{b,-1}^z - \lambda_{32} \cdot \left(r_{b,-1}^f + \frac{\Delta xr^f}{xr^f} \right) - \lambda_{33} \cdot r_{m,-1}^z - \lambda_{34} \cdot \frac{YD^z}{V^z} + \lambda_{35} \cdot r_{e,-1}^z + \\ - \lambda_{36} \cdot \left(r_{e,-1}^f + \frac{\Delta xr^f}{xr^f} \right) \tag{55}$$

$$\frac{E_{h,z}^f}{V^z} = \lambda_{40} - \lambda_{41} \cdot r_{b,-1}^z - \lambda_{42} \cdot \left(r_{b,-1}^f + \frac{\Delta xr^f}{xr^f} \right) - \lambda_{43} \cdot r_{m,-1}^z - \lambda_{44} \cdot \frac{YD^z}{V^z} - \lambda_{45} \cdot r_{e,-1}^z + \\ + \lambda_{46} \cdot \left(r_{e,-1}^f + \frac{\Delta xr^f}{xr^f} \right) \tag{56}$$

where λs are all positive coefficients.[10]

In each area, households' demand for cash is proportional to their expected consumption expenditures (proxied by past consumption):

$$H_h^z = \lambda_c^z \cdot c_{-1}^z \cdot p_{A,-1}^z \tag{57}$$

Households' demand for personal loans is driven by their purchases of durable goods and their consumption in excess of disposable income:

$$L_h^z = L_{h,-1}^z \cdot (1 - \psi_1^z) + \max \left[c^z \cdot p_A^z - YD^z, \psi_2^z \cdot \Delta(\mathbf{p}^{zT} \cdot \mathbf{dc}^z) \right] \tag{58}$$

[10] Note that λs are defined in such a way that: *a*) horizontal constraints on coefficients of rates of interest/return for each financial asset are met; *b*) vertical constraints for cross-asset coefficients of rates of interest/return are met; and *c*) the sum of autonomous shares of assets to total wealth (additional vertical constraints) is lower than unity, because households can hold cash and bank deposits in addition to government bills and corporate equity (see Godley and Lavoie 2007, sections 5.6.2 and 5.6.3[5]). These constraints must be verified at the global level.

where ψ_1^z is the share of (past) loans repaid in each period, ψ_2^z is the share of new durable goods funded by bank loans, and $\mathbf{dc}^z$ is the vector defining the real stocks of durable goods (we refer to subsection 2.12, equation 92).

In each area, bank deposits are the buffer stock of domestic investors:

$$M_h^z = V^z + L_h^z - H_h^z - B_{h,z}^z - B_{h,z}^f - E_{h,z}^z - E_{h,z}^f \tag{59}$$

2.8 Price setting and production function

Private firms use a markup rule. More precisely, they set industry-specific costing margins over their unit costs of production, including fixed capital costs. The vector of unit prices of reproduction is:

$$\mathbf{p}^{z*} = \mathbf{w}^z \odot \mathbf{l}^z + \mathbf{p}^{z*} \cdot \mathbf{A} \odot \mathbf{m}^{z*} \odot \mathbf{h}_d^z \tag{60}$$

where $\mathbf{m}^{z*} = \{1 + \mu_j^{z*}\}$ is the vector of normal mark-ups and $\mathbf{h}_d^z = \{1 + h_j^z \cdot \delta^z\}$ is the vector of the portions of fixed capital that are being amortized in each period,[11] from which one obtains:

$$\mathbf{p}^{z*} = \begin{pmatrix} p_1^{z*} \\ p_2^{z*} \\ \vdots \\ p_5^{z*} \end{pmatrix} = \begin{pmatrix} \frac{w_1^z}{pr_1^z} + (p_1^{z*} \cdot a_{11} + p_2^{z*} \cdot a_{21} + \cdots + p_5^{z*} \cdot a_{51}) \cdot (1 + \mu_1^{z*}) \cdot (1 + h_1^z \cdot \delta^z) \\ \frac{w_2^z}{pr_2^z} + (p_1^{z*} \cdot a_{12} + p_2^{z*} \cdot a_{22} + \cdots + p_5^{z*} \cdot a_{52}) \cdot (1 + \mu_2^{z*}) \cdot (1 + h_2^z \cdot \delta^z) \\ \vdots \\ \frac{w_5^z}{pr_5^z} + (p_1^{z*} \cdot a_{15} + p_2^{z*} \cdot a_{25} + \cdots + p_5^{z*} \cdot a_{55}) \cdot (1 + \mu_5^{z*}) \cdot (1 + h_5^z \cdot \delta^z) \end{pmatrix}$$

While this resembles Sraffa (1960)[7]), both wage rates and normal mark-ups are allowed to differ across industries here. In other words, we assume no tendency for industry-specific wage and profit rates to level out.

In each industry, potential output is simply defined as a direct, linear, function of the available labour force:

$$\mathbf{x}^{z*} = \mathbf{pr}^z \odot \mathbf{POP}^z \tag{61}$$

Actual market prices grow above (or fall below) reproduction prices if actual outputs exceed (or are lower than) potential outputs.[12] Besides, they include VAT rates and tariffs on imports:

$$\mathbf{p}^z = \left[\mathbf{p}^{z*} + \Gamma_x^z \odot (\mathbf{x}_{-1}^z - \mathbf{x}_{-1}^{z*})\right] \odot \left[\begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} + \tau_{vat}^z + \tau_{tar}^f\right] \tag{62}$$

where Γ_x^z is a vector of positive coefficients defining the sensitivity of market prices to output gaps.

The average price level faced by domestic households depends on the basket of goods they consume in each period:

$$p_A^z = \mathbf{p}^{zT} \cdot \beta^z \tag{63}$$

Similarly, the average price paid by production firms to buy investment goods is:

$$p_I^z = \mathbf{p}^{zT} \cdot \iota \tag{64}$$

[11] We refer again to subsection 2.2.

[12] It follows that actual marks-ups fall below normal mark-ups as long as $p_j^z < p_j^{z*}$, and they exceed them as long as $p_j^z > p_j^{z*}$, $\forall j = 1, 2, ..., 5$.

The average price paid by the government is:

$$p_G^z = \mathbf{p}^{zT} \cdot \sigma \tag{65}$$

Finally, the average price of import of final goods is:

$$p_M^z = xr^f \cdot \mathbf{p}^{fT} \cdot \eta \tag{66}$$

Notice that these average prices are used to express each component of aggregate demand in real terms, thus avoiding using disaggregated functions for consumption, investment, government spending and foreign trade.

2.9 The balance of payments

In each area, real import of final goods is defined by a logarithm function of both the international price gap and the real domestic disposable income:

$$\log(imp^z) = m_0^z - m_1^z \cdot \left[\log(p_{M,-1}^z) - \log(p_{A,-1}^z)\right] + m_2^z \cdot \log\left(\frac{YD_{-1}^z}{p_{A,-1}^z}\right) \tag{67}$$

where $m_0^z < 0$, $m_1^z > 0$, and $m_2^z > 0$.

Nominal import of final goods is:

$$IMP^z = p_M^z \cdot imp^z \tag{68}$$

The volume of export of final goods to the other area is:

$$exp^z = imp^f \tag{69}$$

Nominal export of final goods is:

$$EXP^z = xr^f \cdot IMP^f \tag{70}$$

The trade balance of each area is:

$$TB^z = EXP^z - IMP^z \tag{71}$$

The current account balance is:

$$\begin{aligned} CAB^z = TB^z &+ r_{b,-1}^f \cdot B_{s,z,-1}^f \cdot xr_{-1}^f - r_{b,-1}^z \cdot B_{s,f,-1}^z + r_{b,-1}^f \cdot B_{cb,s,z,-1}^f \cdot xr_{-1}^f + \\ &+ xr^f \cdot (1-\omega^f) \cdot FF^f \cdot \frac{E_{s,z,-1}^f}{E_{s,-1}^f} - (1-\omega^z) \cdot FF^z \cdot \frac{E_{s,f,-1}^z}{E_{s,-1}^z} \end{aligned} \tag{72}$$

The financial account balance, net of official transactions, is:

$$KABP^z = \Delta B_{s,f}^z - xr^f \cdot \Delta B_{s,z}^f + \Delta E_{s,f}^z - xr^f \cdot \Delta E_{s,z}^f \tag{73}$$

Finally, the net accumulation of financial assets in each area is:

$$NAFA^z = DEF_g^z + CAB^z \tag{74}$$

2.10 Exchange rate regimes

As mentioned, exchange rates are quoted indirectly, that is, the exchange rate is the price of one unit of domestic currency expressed in foreign currency. Obviously, the exchange rate of the foreign area is the reciprocal of the exchange rate of the domestic area:

$$xr^f = \frac{1}{x^z} \tag{75}$$

Following Godley and Lavoie (2007, section 12.4, [5]), central bank's holdings of government bills are modelled asymmetrically. The amount of domestic government bills held by the domestic central bank is obtained as an accounting identity from column 7 of the transactions-flow matrix (Table 2, changes in stocks):

$$\Delta B^z_{cb,z} = \Delta H^z_s - \Delta A^z_s - xr^f \cdot \Delta B^f_{cb,s,z} \tag{76}$$

Conversely, column 12 of the balance sheet matrix (Table 1) provides the following identity (vertical constraint) for the other area's central bank:

$$B^f_{cb,f} = H^f_s - A^f_s \tag{77}$$

The balance sheet of the central bank in the first area comprises domestic government bills, foreign government bills, and advances to commercial banks as its assets. On the liability side, cash is the primary component.[13] The balance sheet of the central bank in the second area is similar, but it is assumed that it does not hold government bills issued in the first area.

We consider two different exchange rate regimes: a fixed exchange rate, and a (quasi) floating exchange rate.

2.10.1 Fixed exchange rate

Under the fixed exchange rate regime, the supply of foreign government bills to domestic households is defined as:

$$\Delta B^f_{s,z} = xr^z \cdot B^f_{h,z} \tag{78}$$

The supply of government bills of the second area to the central bank of the first area is:

$$B^f_{cb,s,z} = B^f_s - B^f_{s,z} - B^f_{s,f} - B^f_{cb,f} - B^f_b \tag{79}$$

Therefore, the hidden or redundant equation is the one that matches the amount of domestic government bills held by the domestic central bank with the horizontal constraint (in terms of cross-sector holdings of bills) defined by the balance sheet matrix:

$$B^z_{cb,z} = B^z_s - B^z_{s,z} - B^z_{s,f} - B^z_b \tag{80}$$

The accounting structure of the model is now complete. However, a few additional model features have been included to allow for a broader range of experiments, which are discussed below.

2.10.2 Quasi-floating exchange rate

In the alternative regime, the exchange rate is allowed to adjust gradually to reflect the relative demand for national currencies:

$$\Delta xr^z = \chi \cdot \frac{CAB^z_{-1}}{YN^z_{-1}} \tag{81}$$

where χ is a positive parameter defining the speed of adjustment of the exchange rate to the current account balance to total value added ratio. As a result, the domestic currency keeps appreciating (depreciating) as long as the area runs into current account surpluses (deficits).

Note that while the mechanism above increases (reduces) the value of the amount of foreign government bills supplied to domestic households (via equation 78), the domestic central bank is still buying foreign government bills (via 79), albeit in a lower (higher) amount compared with that purchased under a fixed exchange rate regime.[14]

[13] For the sake of simplicity, we assume away bank reserves.

[14] In this scenario, the domestic central bank should be purchasing all the unsubscribed foreign bills to maintain exchange rate stability.

Table 1: Balance-sheet matrix in period 20 (current prices, Area 1 currency)

	Area 1						Area 2					
	H	F	G	B	CB	xr	H	F	G	B	CB	Tot
Money	74.31				-74.31	1	74.31				-74.31	0.00
Advances				0.00	0.00	1				0.00	0.00	0.00
Deposits	444.09			-444.09		1	444.09			-444.09		0.00
Loans	-14.66	-95.86		110.53		1	-14.66	-95.86		110.53		0.00
Area 1 bills	27.86		-449.66	333.56	74.31	1	13.93					0.00
Area 2 bills	13.93				0.00	1	27.86		-449.66	333.56	74.31	0.00
Area 1 shares	11.14	-11.70				1	0.56					0.00
Area 2 shares	0.56					1	11.14	-11.70				0.00
Capital stock		107.56				1		107.56				215.13
Net financial wealth	-557.22		449.66			1	-557.22		449.66			-215.13
Total	0.00	0.00	0.00	0.00	0.00		0.00	0.00	0.00	0.00	0.00	0.00

Table 2: Transactions-flow matrix in period 20 (current prices, Area 1 currency)

	Area 1							Area 2						
	H	F (y)	F (k)	G	B	CB	xr	H	F (y)	F (k)	G	B	CB	Tot
Consumption	-745.07	745.07					1	-745.07	745.07					0.00
Investment		8.04	-8.04				1		8.04	-8.04				0.00
Government spending		183.87		-183.87			1		183.87		-183.87			0.00
Export of Area 1		28.78					1		-28.78					0.00
Import of Area 1		-28.78					1		28.78					0.00
[Value added]		[922.09]					1		[922.09]					0.00
Wage bill	432.49	-432.49					1	432.49	-432.49					0.00
Corporate profit	480.64	-480.64	0.00				1	480.64	-480.64	0.00				0.00
Amortization		-5.24	5.24				1		-5.24	5.24				0.00
Bank profit	8.83				-8.83		1	8.83				-8.83		0.00
CB profit				2.96		-2.96	1				2.96		-2.96	0.00
Income tax revenue	-184.73			184.73			1	-184.73			184.73			0.00
VAT revenue		-14.61		14.61			1		-14.61		14.61			0.00
Tariffs revenue		-0.28		0.28			1		-0.28		0.28			0.00
Interests on deposits	8.83				-8.83		1	8.83				-8.83		0.00
Interests on loans	-0.55	-3.72			4.27		1	-0.55	-3.72			4.27		0.00
Interests on Area 1 bills	1.11			-18.01	13.39	2.96	1	0.56						0.00
Interests on Area 2 bills	0.56					0.00	1	1.11			-18.01	13.39	2.96	0.00
Change in money stock	-0.29					0.29	1	-0.29					0.29	0.00
Change in advances					0.00	0.00	1					0.00	0.00	0.00
Change in deposits	-2.52				2.52		1	-2.52				2.52		0.00
Change in loans	0.90		2.76		-3.66		1			2.76		-3.66		0.00
Change in Area 1 bills	-0.11			-0.70	1.14	-0.29	1	-0.05						0.00
Change in Area 2 bills	-0.05					0.00	1	-0.11			-0.70	1.14	-0.29	0.00
Change in Area 1 shares	-0.04		0.04				1							0.00
Change in Area 2 shares							1	-0.04		0.04				0.00
Revaluation effects	0.00	0.00	0.00	0.00	0.00	0.00	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Total	0.00	0.00	0.00	0.00	0.00	0.00		0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 3: Multi-area input-output matrix in period 20 (current prices, Area 1 currency)

	Area 1 demand for inputs					Area 2 demand for inputs						
	M	A	S	W	R	M	A	S	W	R	Final dem.	Output
Area 1 production												
Manufacturing	67.01	66.89	67.13	26.13	0.00	5.58	5.57	5.59	2.18	0.00	312.33	558.43
Agriculture	67.01	66.89	67.13	26.13	0.00	5.58	5.57	5.59	2.18	0.00	311.35	557.45
Services	67.01	66.89	67.13	26.13	0.00	5.58	5.57	5.59	2.18	0.00	313.31	559.41
Waste manag.	67.00	66.89	67.12	0.00	0.00	5.58	5.57	5.59	0.00	0.00	0.00	217.76
Recycling	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Area 2 production												
Manufacturing	5.58	5.57	5.59	2.18	0.00	67.01	66.89	67.13	26.13	0.00	312.33	558.43
Agriculture	5.58	5.57	5.59	2.18	0.00	67.01	66.89	67.13	26.13	0.00	311.35	557.45
Services	5.58	5.57	5.59	2.18	0.00	67.01	66.89	67.13	26.13	0.00	313.31	559.41
Waste manag.	5.58	5.57	5.59	0.00	0.00	67.00	66.89	67.12	0.00	0.00	0.00	217.76
Recycling	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Value added												
~ Compensation of employees	128.65	128.41	128.89	46.55	0.00	128.65	128.41	128.89	46.55	0.00		
~ G.O. surplus and mixed incomes	139.41	139.18	139.64	86.27	0.00	139.41	139.18	139.64	86.27	0.00		
Taxes on production	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		
Output	558.43	557.45	559.41	217.76	0.00	558.43	557.45	559.41	217.76	0.00		

2.11 Waste and emissions

In each area, waste accumulates as goods and services are produced. The waste associated with each domestic industry is calculated using the related waste to output ratio, ζ_j^z, that is:

$$wa_j^z = wa_{j,-1}^z + x_j^z \cdot \zeta_j^z - x_j^z \cdot a_{5,j} \tag{82}$$

$\forall j = 1, 2, ..., 5$, where the last component $(x_j^z \cdot a_{5,j})$ shows that, in principle, waste can be reduced by recycling it and using is as an input for the other industries.

Total domestic waste (net of recycling) is therefore:

$$wa^z = \sum_{j=1}^{4} wa_j^z \tag{83}$$

If one assumes away land emissions, annual emissions of CO_2 can be calculated for each industry by multiplying their respective output by the industry-specific energy intensity coefficient ($\varepsilon_j^z = Ej_j^z / x_j^z$)), the industry-specific share of non-renewable energy $(1 - \eta_{en,j}^z)$, and a uniform CO_2 intensity coefficient ($\beta_e^z = Gt/Ej$). Emissions linked with each domestic industry are:

$$emis_j^z = x_j^z \cdot (1 - \eta_{en,j}^z) \cdot \varepsilon_j^z \cdot \beta_e^z \tag{84}$$

Therefore, total domestic emissions per year are:

$$emis^z = \mathbf{x}^{zT} \cdot \left\{ \left[\begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} - \eta_{en}^z \right] \odot \varepsilon^z \right\} \cdot \beta_e^z = \sum_{j=1}^{5} emis_j \tag{85}$$

where η_{en}^z is the vector of industry-specific renewable energy percentages.

In each area, cumulative co_2 emissions are:

$$co_2^z = co_{2,-1}^z + emis^z \tag{86}$$

Atmospheric temperature is simply calculated as a function of CO_2 concentration at the global level:

$$temp = \frac{1}{1 - fnc} \cdot tcre \cdot (co_2^z + co_2^f) \tag{87}$$

where fnc is the non-CO_2 fraction of total anthropocentric forcing, and $tcre$ is the transient climate response to cumulative carbon emissions.

2.12 Matter extraction and energy use

In each area, the material contents of outputs can be defined using the corresponding vector of industry-specific matter-intensity coefficients, ϕ^z, that is:

$$x_{mat}^z = \phi^{zT} \cdot \mathbf{x}^z \tag{88}$$

The quantity of matter actually extracted in each period also depends on recycling:

$$mat^z = x_{mat}^z - rec^z \tag{89}$$

Both the socioeconomic stock and industrial waste can be (partially) recycled:

$$rec^z = \rho_{dis}^z \cdot dis^z + q_5^z \cdot x_5^z \tag{90}$$

Figure 1: Sankey diagram of cross-sector transactions and changes in stocks in $t = 10$

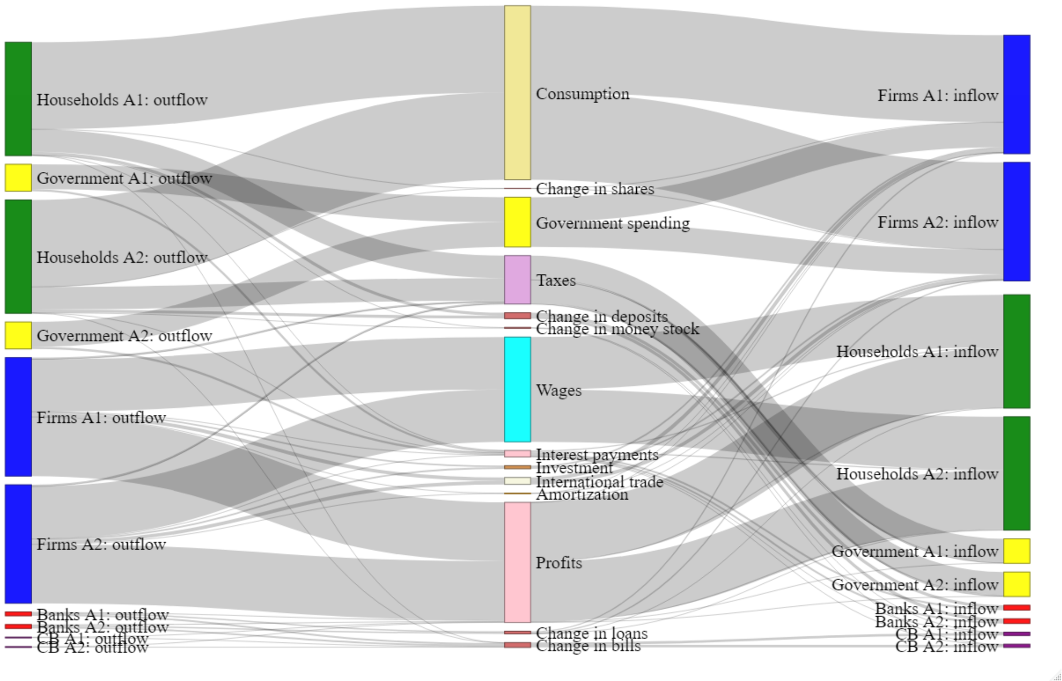
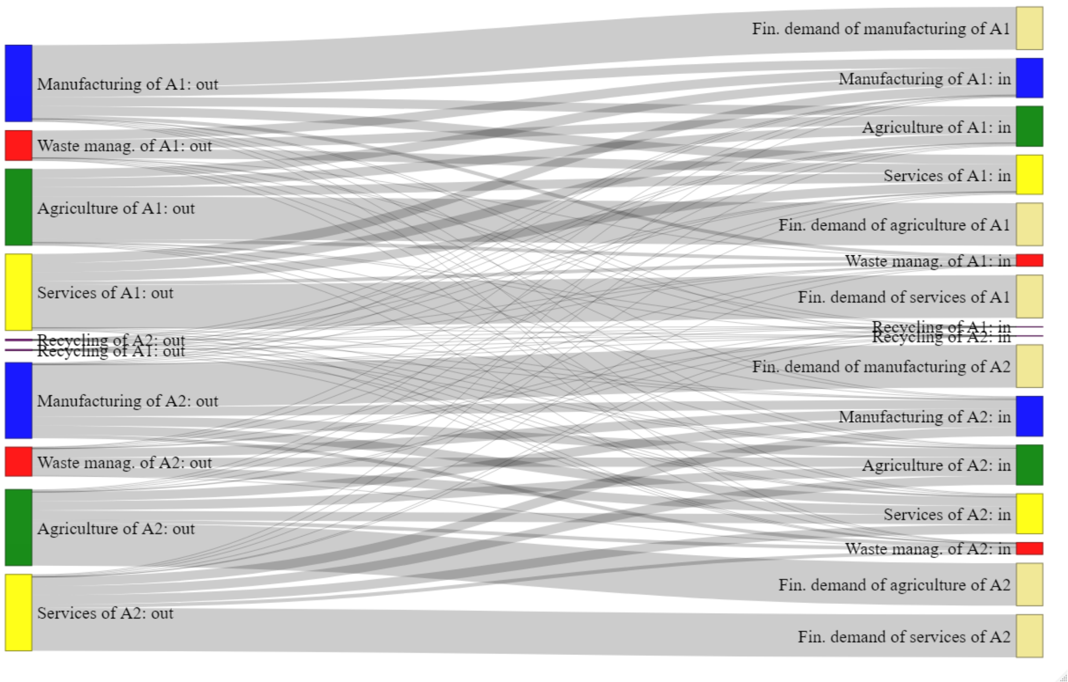


Figure 2: Sankey diagram of cross-industry interdependencies in $t = 10$



where dis^z is the discarded socioeconomic stock, ρ^z_{dis} is the associated rate of recycling, and $q^z_5 \cdot x^z_5$ is the matter content of the recycling industry's output.

The discarded socioeconomic stocks is:

$$dis^z = \phi^{zT} \cdot (\zeta^z_{dc,-1} \odot \mathbf{dc}^z_{-1}) \tag{91}$$

where ζ^z_{dc} is vector of the percentages of durable consumption goods discarded every year by product/industry.

New durable goods equal all produced goods minus discarded goods:

$$\Delta \mathbf{dc}^z = \beta^z \cdot c^z - \zeta^z_{dc,-1} \odot \mathbf{dc}^z_{-1} \tag{92}$$

Finally, the socioeconomic stock accumulates as new material goods are produced and reduces as a share of those goods is discarded every year:

$$\Delta k^z_h = x^z_{mat} - dis^z \tag{93}$$

Like material contents, the energy contents of outputs can be defined using the corresponding vector of industry-specific intensity coefficients, ε^z, that is:

$$en^z = \varepsilon^{zT} \cdot \mathbf{x}^z \tag{94}$$

Renewable energy is just a share of total energy used in each industry:

$$en^z_R = \varepsilon^{zT} \cdot \eta^z_{en} \odot \mathbf{x}^z \tag{95}$$

Non-renewable energy is therefore:

$$en^z_N = en^z - en^z_R \tag{96}$$

We can now calculate the global stocks of matter and energy. The annual change in the stock of material reserves is:

$$\Delta k_{mat} = conv_{mat} - mat^z - mat^f \tag{97}$$

Material resources converted into reserves are:

$$conv_{mat} = \sigma_{mat} \cdot res_{mat} \tag{98}$$

where σ_{mat} is the speed of conversion and res_{mat} is the quantity of resources, which reduce as more resources are converted into reserves:

$$res_{mat} = res_{mat,-1} - conv_{mat} \tag{99}$$

Similarly, the equations defining energy depletion are:

$$\Delta k_{en} = conv_{en} - en^z_N - en^f_N \tag{100}$$

$$conv_{en} = \sigma_{en} \cdot res_{en} \tag{101}$$

$$res_{en} = res_{en,-1} - conv_{en} \tag{102}$$

where σ_{en} is the speed of conversion of energy resources into reserves.

2.13 Circular economy innovations

The label “circular economy” (CE) denotes a set of policies and practices that aim at reusing, repairing, sharing, and recycling products and resources to create a closed-loop system, thus minimizing waste, pollution and CO_2 emissions.[15] A simple way to introduce a CE innovation in the model above is to consider a 5-industry economy, in which the first four

[15] For a thorough discussion on the definition of CE, see Bimpizas-Pinis et al. 2021.[1]

industries produce standard goods and services and waste management, whereas the fifth industry deals with waste recycling.

As long as waste is not recycled, the matrix of technical coefficients is:

$$
\mathbf{A} = \left(\begin{array}{ccccc||ccccc}
a_{11} & a_{12} & a_{13} & a_{14} & 0 & a_{16} & a_{17} & a_{18} & a_{19} & 0 \\
a_{21} & a_{22} & a_{23} & a_{24} & 0 & a_{26} & a_{27} & a_{28} & a_{29} & 0 \\
a_{31} & a_{32} & a_{33} & a_{34} & 0 & a_{36} & a_{37} & a_{38} & a_{39} & 0 \\
a_{41} & a_{42} & a_{43} & a_{44} & 0 & a_{46} & a_{47} & a_{48} & a_{49} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
a_{61} & a_{62} & a_{63} & a_{64} & 0 & a_{66} & a_{67} & a_{68} & a_{69} & 0 \\
a_{71} & a_{72} & a_{73} & a_{74} & 0 & a_{76} & a_{77} & a_{78} & a_{79} & 0 \\
a_{81} & a_{82} & a_{83} & a_{84} & 0 & a_{86} & a_{87} & a_{88} & a_{89} & 0 \\
a_{91} & a_{92} & a_{93} & a_{94} & 0 & a_{96} & a_{97} & a_{98} & a_{99} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right)
$$

All industries generate waste, but no waste is used as input in the domestic economy ($a_{51} = a_{52} = a_{53} = a_{54} = 0$) or in the foreign economy ($a_{106} = a_{107} = a_{108} = a_{109} = 0$). Additionally, no inputs are used in the waste recycling industry of the domestic economy ($a_{15} = a_{25} = a_{35} = a_{45} = 0$) or of the foreign economy ($a_{610} = a_{710} = a_{810} = a_{910} = 0$).

The introduction of a simple CE innovation in the domestic economy implies a change in technical coefficients such that the new matrix is:

$$
\mathbf{A'} = \left(\begin{array}{ccccc||ccccc}
a'_{11} \le a_{11} & a'_{12} \le a_{12} & a'_{13} \le a_{13} & a'_{14} \le a'_{14} & a'_{15} > 0 & a_{16} & a_{17} & a_{18} & a_{19} & 0 \\
a'_{21} \le a_{21} & a'_{22} \le a_{22} & a'_{23} \le a_{23} & a'_{24} \le a'_{24} & a'_{25} > 0 & a_{26} & a_{27} & a_{28} & a_{29} & 0 \\
a'_{31} \le a_{31} & a'_{32} \le a_{32} & a'_{33} \le a_{33} & a'_{34} \le a'_{34} & a'_{35} > 0 & a_{36} & a_{37} & a_{38} & a_{39} & 0 \\
a'_{41} \le a_{41} & a'_{42} \le a_{42} & a'_{43} \le a_{43} & a'_{44} \le a'_{44} & a'_{45} > 0 & a_{46} & a_{47} & a_{48} & a_{49} & 0 \\
a_{51} > 0 & a_{52} > 0 & a_{53} > 0 & a_{54} > 0 & 0 & a_{56} & a_{57} & a_{58} & a_{59} & 0 \\
\hline
a'_{61} \le a_{61} & a'_{62} \le a_{62} & a'_{63} \le a_{63} & a'_{64} \le a'_{64} & a'_{65} > 0 & a_{66} & a_{67} & a_{68} & a_{69} & 0 \\
a'_{71} \le a_{71} & a'_{72} \le a_{72} & a'_{73} \le a_{73} & a'_{74} \le a'_{74} & a'_{75} > 0 & a_{76} & a_{77} & a_{78} & a_{79} & 0 \\
a'_{81} \le a_{81} & a'_{82} \le a_{82} & a'_{83} \le a_{83} & a'_{84} \le a'_{84} & a'_{85} > 0 & a_{86} & a_{87} & a_{88} & a_{89} & 0 \\
a'_{91} \le a_{91} & a'_{92} \le a_{92} & a'_{93} \le a_{93} & a'_{94} \le a'_{94} & a'_{95} > 0 & a_{96} & a_{97} & a_{98} & a_{99} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right)
$$

In short, the CE innovation entails a reduction in the quantities of products and services used as inputs in the same industries. This is possible because recycled waste now enters their production processes.[16] Besides, outputs from other industries are used as inputs in the waste recycling industry.

The unit price of recycled waste now enters equation 60 in subsection 2.8. It is defined in the same way as the other prices. The mark-up applied by the recycling industry is set using the average mark-up of the economy:

$$
\mu_5^z = \mu_{5,-1}^z + \gamma_\mu^z \cdot (\bar{\mu}^z - \mu_{5,-1}^z), \quad with: \bar{\mu}^z = \frac{\sum_{j=1}^{4} \mu_j^z}{4} \tag{103}
$$

where γ_μ^z is the speed of convergence of the initial mark-up value (0 in the baseline scenario) to the average one.

This model assumes that technical change (that is, the value of a'_{ij}) is set by the policy makers, while the average speed of convergence of technical coefficients to their target values is defined as a linear, positive function of government expenditures.

[16] As CE innovation seems to imply some degree of input substitutability, one might notice that *smooth substitutability*, within the *same production function*, is one of the key assumptions of neoclassical general equilibrium models. However, input substitution is only possible here because of a *change in the techniques of production*.

Focusing on the domestic economy (that is, on the first five columns of matrix $\mathbf{A}'$), each coefficient is defined as:

$$a_{ij} = a_{ij,-1} + \gamma_A^z \cdot (a'_{ij,-1} - a_{ij,-1}) \tag{104}$$

$\forall\ i = 1, 2, ..., 10$ and $j = 1, 2, ..., 5$, where γ_A^z is the average speed of transition towards a (partial) CE production system, which is defined as:

$$\gamma_A^z = \gamma_{A0}^z + \Gamma_A^{zT} \cdot \sigma^z \cdot gov_{-1}^z \tag{105}$$

where γ_{A0}^z is a positive scalar, whereas $\Gamma_A^z = \{\gamma_{Aj}^z\}$ is the vector that defines the industry-specific sensitivities (of the speeds of adjustment) to government final demands.[17]

[17] Notice that: $\sigma^z \cdot gov^z = \begin{pmatrix} \sigma_1^z \\ ... \\ \sigma_5^z \end{pmatrix} \cdot gov^z = \begin{pmatrix} \sigma_1^z \cdot gov^z \\ ... \\ \sigma_5^z \cdot gov^z \end{pmatrix}$.

References

[1] Bimpizas-Pinis, M., Bozhinovska, E., Genovese, A., Lowe, B., Pansera, M., Alberich, J.P. and Ramezankhani, M.J. (2021) “Is efficiency enough for circular economy?”. *Resources, Conservation & Recycling*, 167: 105399.

[2] Carnevali, E. (2022) “A new, simple SFC open economy framework”. *Review of Political Economy*, 34(3): 504-533.

[3] Carnevali, E., Deleidi, M., Pariboni, R. and Veronese Passarella, M. (2019) “SFC dynamic models: features, limitations and developments”. In: P. Arestis and M. Sawyer (eds.), *Frontiers of Heterodox Economics, Series: International Papers in Political Economy*, Basingstoke & New York: Palgrave Macmillan, pp. 223-276.

[4] Codina, O. V. and Fevereiro, J. B. R. T. (2022) “Macroeconomic Scenario Exploration. Cheat Sheet: Representation of the economy via Input-Output tables”, presented at *JUST2CE Consortium Meeting*, Thessaloniki, 1 July 2022.

[5] Godley, W. and Lavoie, M. (2007) *Monetary economics: an integrated approach to credit, money, income, production and wealth.* Springer.

[6] Rivera, G.L., Malliet, P., Saussay, A. and Reyns, F. (2018) “The State of Applied Environmental Macroeconomics”. *Revue de l’OFCE*, 157 (3): 133-149.

[7] Sraffa, P. (1960) *Production of commodities by means of commodities. Prelude to a critique of economic theory.* Vora & co.

[8] Waugh, F.V. (1950) Inversion of the Leontief matrix by power series. *Econometrica*, 18(2): 142-154.

[9] Zink, T. and Geyer, R. (2017) “Circular Economy Rebound”. *Journal of Industrial Ecology*, 21: 593-602.