

Expectations and the stability of stock-flow consistent models

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Abstract

Expectations are usually introduced in macroeconomic stock-flow consistent models (SFC-models from hereon) in an ad hoc way, without much motivation. Moreover, these are usually very simple forms of expectations, and certainly not some form of rational expectations. The implicit assumption is that expectations do not matter very much in these models.

In this paper, we argue that expectations are very important in understanding the way the economy reacts to a shock, since the stability of the economy is dependent on the nature of expectations. We show for instance that for simple expectations, the more backward-looking they are, the more stable the economy tends to become, whereas the opposite is true for enhanced expectations. For that purpose, we use a simple model, based on the models in Godley & Lavoie, 2007. The model includes next to households, firms and government, a financial sector and a foreign sector.

First, we analyse the stationary state solution and analyse its properties. We show that this model is only stable when either the tax rate or government debt is not too high. We also point out the self-fulfilling properties of optimism and pessimism in expectations in this model. Next, we show that under “perfect foresight” the model becomes saddle point stable – strong restrictions on taxes and government debt are necessary to guarantee the stability of the model. Under simple naïve expectations the model becomes more stable compared to the stationary state. However, it becomes less stable when naïve expectations are enhanced by the impact of growth. Finally, we show how under fundamentalist and adaptive expectations the stability of the model is affected in an intermediate way.

Second, we analyse the adaption process after a shock. Here we find that GDP adjusts very slowly under naïve expectations and faster under stationary and fundamentalist expectations.

All in all, we conclude that expectations do matter: the kind of expectations introduced in an SFC model strongly influence both its stability properties and the speed of adjustment to shocks. In our simple model, fundamentalist expectations turn out to have the best combination of stability and speed of adjustment.

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1 Introduction

Expectations are usually introduced in macroeconomic stock-flow consistent models (SFC-models from hereon) in an ad hoc way, without much motivation. Moreover, very simple forms of expectations are used – the implicit assumption is that expectations do not matter very much in these models. Also, rational expectations in one form or another are not consistent with the post-Keynesian tradition of SFC-models. However, the way expectations are modelled in SFC-models is very important for three reasons.

The first reason is that expectations influence the way the economy reacts to a shock. As we elaborate below, the stability of the economy and the room for stabilising fiscal policy is dependent on the nature of expectations. We show for instance that the more backward-looking expectations are, the more stable the economy tends to become and the larger the room for fiscal policy is.

The second reason is that expectations themselves can also be a source of shocks. We show how optimism or pessimism in expectations can lead to self-fulfilling prophecies. Moreover, one may wonder what happens when persons have different expectations under different circumstances and what causes the transition from one type of expectations to another. Although we do not elaborate that point here, the different types of expectations we explore provide already an interesting insight in the potential implications of this phenomenon.

The third reason to look at expectations in SFC models lies in the nature of these models themselves. The way shocks affect an economy is dependent on the buffers the economy has available to absorb these shocks – this also holds for both expected shocks and shocks in expectations. It has only been recently recognised in macroeconomic modelling that wealth accumulation, together with debt accumulation, plays an important role – sufficient wealth accumulation is required to deal with shocks. As we elaborate below, it is very natural to use SFC-models to study wealth accumulation. These models pay explicit attention to the role of both monetary and fiscal policy, moreover they recognise explicitly the role played by financial institutions. See Godley and Lavoie (2007) for a seminal introduction.

In our analysis we use a simple macro model in the SFC tradition, to keep it analytically tractable. In the tradition of Godley and Lavoie (G&L from hereon), the model is typically Keynesian: product markets clear through quantity adjustments, and the model is demand-led. In spite of its simplicity, we show how introducing different types of expectations in this model leads to new insights in the way expectations affect the stability of the model.

To illustrate the impact of expectations on the stability of an economy we present a simple model in section 2, based on the models in G&L. In order to keep the analysis as simple as possible, we abstain from economic growth and stick to a stationary economy. Next to firms and households, the model includes both government and a foreign sector to allow for a better understanding of the multiplier impact of shocks to the economy and the buffer role played by government bonds and foreign reserves. We also include a banking sector, since this plays an important intermediary role in the accumulation and distribution of financial wealth over buffers. Moreover banks provide an additional buffer to the economy in the form of deposits.

In section 3 we analyse the stationary state of the model and analyse its properties. Moreover, we show that with stationary expectations this model is only stable when both taxes and government debt are not too high (and there is a trade-off between both). We also point out the self-fulfilling properties of optimism and pessimism in expectations in this model.

We then analyse the impact of different forms of expectations in section 4. We show that under “perfect foresight” the model becomes saddle point stable – strong restrictions on taxes and government debt are necessary to guarantee stability of the model compared to the stationary situation. The intuition is that “perfect foresight” intensifies the impact of shocks in the economy, inducing self-fulfilling prophecies. On the other hand, under simple naïve expectations the model becomes more stable. The intuition is that the path dependency of naïve expectations hampers the adjustments to shocks in the economy and thus mitigates their destabilising impact. However, the model becomes less stable when naïve expectations are enhanced by the impact of growth. We also introduce the notion of fundamentalist expectations and show that these also affect the stability of the model in a positive way, compared to stationary expectations. Finally, we investigate the properties of adaptive expectations and show that these have a similar impact on stability as fundamentalist expectations.

We conclude the discussion of policy rules by comparing in section 5 the properties of expectation rules under the lens of adjustment to economic shocks. We show that, consistently with economic literature, forward looking rules often lead to a faster adjustment to shocks than backward looking, naïve rules. However, fundamentalist expectations turn out to accommodate the adjustment process also quite well.

We present our concluding remarks in section 6.

2 The model

To illustrate the impact of expectations on the stability of an economy we present a simple model in section 2.1. The model includes a financial sector, government and a foreign sector – each of these sectors provides buffers which help to absorb shocks in an economy. We introduce government in our model, since understanding fiscal policy can be very important to contain instabilities in the model and government bonds provide a relevant buffer through household savings. We also introduce a foreign sector in a very simple way to allow for a better understanding of the multiplier impact of shocks to the economy and to analyse the role of foreign reserves in stabilising the economy. In line with the notion of a monetary economy financial wealth is included in our model, in the form of deposits, loans, government bonds and foreign reserves. For that reason the model also is in the tradition of stock-flow consistent modelling as we elaborate in section 2.2. Finally we discuss in section 2.3 the role of expectations in our model, since these are at the core of our analysis. In order to keep the analysis as simple as possible, we abstain from economic growth and stick to a stationary economy.

2.1 A simple model, including a financial sector, government and a foreign sector

The model used in this paper is based on G&L (Ch. 7), to which we add government and a foreign sector.¹

The economy consists of five sectors households, firms, government, banks and foreign. Output of the economy consists of consumption goods C , investment goods I and government goods G , and net exports $X - IM$:

$$Y = C + I + G + X - IM \tag{1}$$

¹ The full model is presented in Appendix A, with references to the corresponding equations of G&L.

Firms produce this output and borrow the amount of money necessary to finance their net-investments from banks at a rate r . That is, borrowing by firms implies an additional amount of loans:

$$\Delta L = I - \delta \cdot K_{-1} \quad \delta > 0 \quad (2)$$

where δ is the rate of depreciation of the capital stock K . Note that we introduce here two stocks in our analysis, the capital stock K and the amount of loans L . From equation (2) one observes that in our analysis stocks are measured at the end of the period, and for simplicity we ignore time subscripts t .²

Next to that, firms pay wages to workers such that the wage bill paid by firms is $WB = W \cdot Y / \lambda$ – here W is the endogenous real wage and λ stands for exogenous labour productivity.

Since there are no credit restrictions and firms can borrow unlimited from banks, we assume that firms do not retain earnings to finance their investment (see also G&L, p.223). Therefore, net savings of firms are zero and firm expenditures equal output, such that the wage bill follows from:

$$Y = WB + r_{-1} \cdot L_{-1} + \delta \cdot K_{-1} \quad (3)$$

Investment behaviour of firms follows from:

$$I = \gamma \cdot (K^e - K_{-1}) + \delta \cdot K_{-1} \quad 0 < \gamma < 1 \quad (4)$$

Here K^e represents the expected capital stock. With respect to the capital stock we note that capacity output is defined by:³

$$Y^c = K / \kappa \quad \kappa > 0 \quad (5)$$

The parameter κ is a constant capital output ratio. The notion of a constant capital output ratio introduces the accelerator in the model as we elaborate below. However, in our stationary economy capacity output does not play a role.

Government levies taxes T from households proportional to output at a rate τ and issues bonds B to households at a rate r such that its expenditures are fully covered. Hence does hold:

$$\Delta B = G - T + r \cdot B_{-1} \quad (6)$$

and an increase in debt implies negative government savings.

$$T = \tau \cdot Y \quad 0 < \tau < 1 \quad (7)$$

Government also imposes a debt criterion such that government debt should always be proportional to output at a rate β :

$$B = \beta \cdot Y \quad \beta > 0 \quad (8)$$

As we elaborate below, government expenditures then are endogenous.

Households receive income from labour and wealth. Their savings are used to buy government bonds and are held at banks as deposits M and abroad as reserves R at a rate r . Hence household wealth consists of bonds, deposits and foreign reserves. In Appendix A we provide a further explanation of the reasons that foreign reserves end up in households, with other financial institutions (for instance pension funds) as intermediaries.

² That is, we write $\Delta L = L - L_{-1}$ instead of $\Delta L_t = L_t - L_{t-1}$ where L_t is the amount of loans at end of period t .

³ See G&L (p. 226, n.4), we normalise the utilisation rate at full capacity to unity.

Wealth accumulation follows from household savings, $YD - C$, where YD is disposable income:

$$\Delta M + \Delta B + \Delta R = YD - C \quad (9)$$

$$YD = WB + r_{-1}M_{-1} + r_{-1}B_{-1} - T \quad (10)$$

Household consumption is given by:

$$C = \alpha_1 YD^e + \alpha_2 (M_{-1} + B_{-1} + R_{-1}) + C^A \quad \alpha_1 > 0, \alpha_2 > 0 \quad (11)$$

with YD^e representing expected disposable income. The constant term C^A represents autonomous consumption.

We assume exports X to be exogenous at a value X^A and imports IM to be proportional to output:

$$IM = \mu \cdot Y \quad \mu > 0 \quad (12)$$

The surplus (deficit) of exports over imports leads to an increase (decrease) of foreign debt in the form of reserves R held by households at a rate r . Thus we find:

$$\Delta R = X^A - \mu \cdot Y + r_{-1}R_{-1} \quad (13)$$

Finally, we include the banking sector: banks provide loans L to firms and hold deposits M provided by households. That is, banks accept all demand for deposits by households and provide all loans demanded by firms at the prevailing interest rate r – this implies that they create money. For simplicity we assume that the interest rate on loans is equal to the interest rate on deposits to households. Since:

$$r_{-1}L_{-1} = r_{-1}M_{-1} \quad (14)$$

does hold, bank savings are zero.

We assume the interest rate r to be exogenous. The price level is constant at a level unity.

The working of the model is explained in more detail below.

2.2 Stock-flow consistency and behavioural assumptions

The stock-flow consistent nature of the model implies that all wealth components (stocks) can be identified in the model as assets and liabilities and that all flows in the model should be consistent with changes in wealth.

Table 1. The balance sheet

	Households	Firms	Banks	Government	Abroad	Total
Deposits	+M		-M			0
Bonds	+B			-B		0
Loans		-L	L			0
Fixed Capital		+K				+K
Reserves	+R				-R	0
Wealth	-Vh	0	0	-Vg	-Va	-Vh-Vg-Va
Total	0	0	0	0	0	0

From the balance sheet presented in Table 1 one observes that household wealth V_h consists of deposits, government bonds and foreign reserves. Firm wealth is zero since the only asset, capital, equals the only liability, loans held at banks. For banks, wealth is zero since deposits from households correspond to loans issued to firms. Government wealth V_g is negative, consisting of government debt in the form of bonds, held by households. Finally, foreign wealth V_a , held abroad, consists of reserves only.

Table 2 The accumulation of savings and investment matrix

	Households	Firms	Banks	Gov.	Abroad	Total
Consumption	-C	+C+G		-G		0
Investment		+I				+I
Net Exports		+(X-M)			-(X - M)	0
Wages	WB	-WB				0
Taxes	-T			+T		0
Cons of fixed cap.		- $\delta.K_{-1}$				- $\delta.K_{-1}$
Interest	+ $r_{-1}.B_{-1}$ + $r_{-1}.M_{-1}$ + $r_{-1}.R_{-1}$	- $r_{-1}.L_{-1}$	+ $r_{-1}.L_{-1}$ - $r_{-1}.M_{-1}$	- $r_{-1}.B_{-1}$	- $r_{-1}.R_{-1}$	0
[Net savings]	[Sh]	[0]	[0]	[Sg]	[Sa]	[Stot]
Δ Loans		$+\Delta L$	$-\Delta L$			0
Δ Deposits	$-\Delta M$		$+\Delta M$			0
Δ Bonds	$-\Delta B$			$+\Delta B$		0
Δ Reserves	$-\Delta R$				$+\Delta R$	0
Δ Capital		$-\Delta K$				$-\Delta K$
Total	0	0	0	0	0	0

The flows and changes in stocks in the model are summarised in Table 2. Consistent with equation (8) savings of households, Sh , are held either as deposits at banks, as government bonds or as reserves. In line with equation (2) net investments are financed by borrowing from banks, hence consistent with equation (3) net savings of firms S_f are zero, and the accumulation of capital ΔK equals additional borrowing from banks ΔL . From equation (14) it follows that banks' savings are zero – these banks ensure that the deposits held by households are equal to the amount of loans provided to firms. The foreign sector (abroad) issues these foreign reserves and these constitute foreign savings. Finally, government savings S_g are negative, since government accumulates debt by issuing bonds to households.

To understand the working of the model it is useful to explain the behaviour of each sector more in detail. If we ignore both government and the foreign sector, we find that households save enough to finance net investment of firms, with banks as intermediaries. The underlying assumptions are that households consume according to the consumption function (11), which contains the income multiplier next to wealth effects. Firms invest according to the investment function (4), which contains the accelerator.⁴ The interaction of multiplier and accelerator then lead to an equilibrium as we analyse below (section 3.1) – this income is stable under certain parameter conditions.

⁴ The notion of the accelerator enters through the specification of the expected capital stock, K^e , as we elaborate below.

An important behavioural assumption is that banks accept all demand for deposits by households and provide all loans demanded by firms at the prevailing interest rate r . As G&L (p. 227) emphasise, “the rate of interest on loans can be set by the banking system at the level that it sees fit. The rate of interest on loans is not the result of a market clearing price mechanism”.⁵

By introducing government in the model we add two interesting features. On the one hand, we provide an alternative opportunity for households to invest their savings. Although the total amount of savings held in government bonds is implied directly by $\beta.Y$, one can argue that these savings are decided upon by choosing the debt-output ratio β . As G&L emphasize (pp. 108-9):

The crucial assumption which is made here, and which will be used time and time again, is that money balances are the element of flexibility in a monetary system of production. Money balances are the buffer that absorbs unexpected flows of funds...

The assumption we make is that the entire amount of unexpected saving will be kept in the form of additional cash money balances. Errors in expectations are entirely absorbed by unexpected fluctuations in money balances. Money balances act as a buffer against mistakes in expectations. Any mistake regarding expected disposable income is entirely absorbed by an equivalent unexpected change in money balances.

This implies that, regardless of whether they are realized or not, households actually invest in bills on the basis of their expectations with respect to disposable income that were made at the beginning of the period. This means that the amount of bills held by households at the end of the period is exactly equal to the amount of bills that were demanded by households at the beginning of the period.

On the other hand, one should realise that the choice of a debt-output ratio β , together with choosing a tax rate τ , implies that government expenditures are endogenous in the model. We show below that the stationary state government expenditures to output ratio, $(G/Y)^*$ is given by $(G/Y)^* = \tau - r.\beta$. By choosing a higher debt-output ratio at a given tax rate, more money is needed for interest payments and hence less money is available for government expenditures. That is the reason why in the stationary state $(G/Y)^*$ decreases when β increases. Similarly, since more money is available for government expenditures when the tax rate increases at a given debt-output ratio, in the stationary state $(G/Y)^*$ increases when τ increases. The stationary state budget deficit is $-r.\beta$.

Finally, the introduction of the foreign sector mitigates the multiplier working of the model due to import leakages. Next to that, the impact of the accumulation or decumulation of foreign reserves due to current account surpluses or deficits registered in our model, appears from Tables 1 and 2, changes the outcome of the model – we further analyse this in section 3.1.

We show below that introducing government and a foreign sector has an impact on the stationary state conditions of the model and also influences the stability conditions of the model. Moreover, introducing government allows us to intervene in the economy when changes in expectations or shocks force the economy out the stability corridor as we discuss section 3.

2.3 The nature of expectations

The role of expectations in macroeconomics has been noted as far back as Keynes (Keynes, 1936). When people take economic actions, their forecasts of the future matter. Keynes famously characterised behaviour as the result of animal spirits, which drive people to act through waves of optimism when facing fundamental uncertainty.⁶ These forecasts affect household decisions (how

⁵ G&L section 7.6 show that the rate of interest affects the real wage in the stationary state, such that a higher rate of interest implies a lower real wage.

⁶ “Even apart from the instability due to speculation, there is the instability due to the characteristic of human nature that a large proportion of our positive activities depend on spontaneous optimism rather than on a mathematical expectation, whether moral or hedonistic or economic. Most, probably, of our decisions to do something positive, the full consequences of which will be drawn out over many days to come, can only be taken

much to consume today versus save for an uncertain future) and investment decisions (when expecting a growing or slowing economy)(Piccillo & Poonpakdee, 2021).

With the theory of rational expectations (RE), the literature addressed the issue of model consistency (Lucas, 1972; Muth, 1961). Under RE, economic agents understand how the world works, and they adjust their predictions in real time with all available information. While this approach allowed economists to formalise the key role of expectations in macroeconomic modelling, it became obvious that RE could not realistically describe the dynamics of people's actual expectations (Frankel & Froot, 1987).

Agents (even expert agents, like professional forecasters, or econometricians) do not have *perfect foresight* (Wieland & Wolters, 2011). Equally relevant, agents' forecasts are dynamic, as they change over time. More fundamentally, individual forecasting errors are not random. Specifically, individuals have several recognizable biases (Williams, 1987). They make clustered errors, where mistakes in the same direction arise together, and where periods of high error volatility alternate with periods of low error volatility (Milani, 2017). They also update using new information non-linearly, with a systematic early under-reaction to new information, followed by a systematic over-reaction. Therefore, boundedly rational expectation approaches have been introduced. The most well known are *adaptive learning* (Evans & Honkapohja, 2001) and theories of behavioural biases, like coordinated *waves of optimism and pessimism* (De Grauwe & Rovira Kaltwasser, 2007; Kurz et al., 2013). Evidence shows that these are better ways to describe belief dynamics empirically (Hommes & Lux, 2013). Additionally, beliefs are also heterogeneous across people (De Grauwe & Ji, 2022; Hommes, 2021). Different people likely use different forecasting rules, whose relative performance varies through time (Brock & Hommes, 1997; Chiarella et al., 2014).

Given this rich and growing literature, studies have been able to identify the rules that are most often used by agents as they forecast. Experiments about "learning to forecast" have robustly identified a robust set of rules, which repeatedly explains individual forecasting behaviour. The rules most often adopted by agents are naïve and adaptive expectations on the one hand, and fundamentalist expectations on the other.

2.3.1 Stability

When applied to macroeconomic models, most boundedly rational expectation rules can impact the stationary state and its stability in a variety of ways. Changes to stability arise especially due to the feedback mechanism inherent to the concept of expectations. Expectations are themselves (appropriately) functions of the state variables in the model since, according to the literature above, agents update their beliefs using some mix of current and past information, and then they use this information to make decisions affecting the world today (Lustenhouwer, 2023; t'Serstevens et al., 2022). For this reason, it is also not unusual for expectations to add new state variables to the dynamic system (Evans & Honkapohja, 2001).

For instance, it is natural that backward looking expectations (like naïve and adaptive expectations) amplify the dynamic model dependence on the past. Often this leads to the stability of the model being affected. In several cases (Hommes, 2021; Kurz et al., 2013) the stability of the bounded rationality system is more constrained compared to the benchmark case of perfect foresight. In several cases, backward looking expectations are inserted in a model only for a fraction of the agents. The other fraction may either hold perfect foresight beliefs, with or without costs (Bask & Proaño,

as a result of animal spirits – of a spontaneous urge to action rather than inaction, and not as the outcome of a weighted average of quantitative benefits multiplied by quantitative probabilities." (Keynes, 1936)

2016; Brock & Hommes, 1997) or that fraction forecasts the market to move toward a fundamental value by an given distance every period, as in the fundamentalist expectation rule (Massaro, 2013). This kind of models displays a rich dynamics, and the global stability of the one steady state is dependent on several conditions (Gomez & Piccillo, 2019).

In the case of dynamic stochastic general equilibrium models, expectations in these models make money non-neutral, without the addition of any other frictions. Therefore the role of the central bank is crucial to keep the system stable. The role of monetary policy becomes then to act by ruling on the impact of expectations, rather than is more common for this literature, to react to exogeneous shocks (Kurz et al., 2015)

However, and despite this rich literature, SFC models until now left only a marginal role to the study of how expectation models affect model dynamics. Moreover, while in the approaches above always some form of dynamic optimisation is involved, SFC models prefer to use simple heuristics. G&L (Ch.3) show that due to the buffer function of money, following from including wealth in the consumption function, their simple model will always tend to a steady state solution (provided the parameter conditions are satisfied), and mistakes in expectations will be corrected eventually.⁷ Of course the adjustment path towards the steady state is affected by the mistakes in expectations. They show this for constant and naïve expectations – in the latter case the value of the variable in the previous period is expected also to prevail in the current period.

In their models of Ch. 4 and Ch. 5 G&L also use either constant, or naïve expectations. In Ch.7 they also use expectations in the form of adapting the previous period value to a target value. Only in Ch.9 and the following chapters adaptive expectations are used. Adaptive expectations have been recently used in several SFC models (Burgess et al., 2016; Caiani et al., 2016; Le Heron, 2011; Meijers & Muysken, 2022; Santos & Zezza, 2008). Recently, a study shows that the steady state of SFC models is affected by different specifications of adaptive expectation rules (Kappes & Milan, 2020).

Given this context, there remains a gap in the literature on the impact of the most widely known expectation rules on SFC models. We explore this question and we find that both the steady state and the conditions for its stability can be strongly affected. Below, we show how the system changes with the chosen rules: naïve, stationary expectations, fundamentalist, “perfect foresight”.

2.3.2 SFC with expectations rules

In our model we introduce expected disposable income YD^e in equation (9) and the expected capital stock K^e in equation (4). It is important to understand that disposable income is a flow and the expected capital stock is a stock. We already mentioned that stocks are measured at the end of the period. That implies that the expected capital stock K^e is the stock at the end of period t , with expectations formed at the beginning of period t . In the same vein, the expected disposable income YD^e , formed at the beginning of period t , refers to the income that will be obtained during period t . It then is consistent, in line with equation (5), to assume with respect to K^e :

$$K^e = \kappa \cdot Y^e \tag{15}$$

In this way all that agents have to forecast is output for period t , which will only be known at the end of the period. Having this forecast allows firms and households to form consistent expectations of

⁷ An important observations is that “money ... acts as a ‘buffer’ when expectations turn out to be incorrect.” (p. 78). That is, “if realised income is above the expected income, households will hold the difference in the form of larger than expected money balances.”(p. 79).

expected capital and expected disposable income, since they are both functions of the same period output. We elaborate this for the most common rules of expectations we mentioned above, under the representative agent assumption.

Stationary expectations

The simplest form of expectations from the point of view of the agents is to hold a stationary expectation. This means that at every period in time the agents will continue to forecast that next period the model consistent stationary state will be achieved⁸:

$$Y^e = Y^* \quad (16S)$$

This is a constant expectation which, insofar as the model does achieve stationary state, correctly reflects also the nature of expectations and the model parameters. In this sense, the rule is consistent with the model. It is also mean-reverting, since it allows the agents to make decisions that are moving away from any trend that can be originated by exogenous shocks. The rule is however not rational since it is clear that after any shock, if agents forecast the stationary income value, they are going to be wrong for the period of time that the model is out of stationary state.

Naïve expectations

In the paragraphs above we referred to realistic expectations, which have been repeatedly confirmed through empirical research in the field and in the lab. The simplest form of realistic expectations that is surely relevant for some of the agents at most of the time is backward looking. Under this type of rules, expectations use past information to forecast unobservable variables. The simplest form of backward looking expectations is given by naïve expectations. Under this rule, income during the current period is expected to be equal to income in last period:

$$Y^e = Y_{-1} \quad (16N)$$

While simple naïve expectations add a friction to the economic system, they do not represent the only way expectations can be backward looking. It is empirically shown, most recently in learning to forecast experiments (Hommes, 2021), that at least a portion of the people at different points in time focusses on the trend of last period change in the unknown variable, rather than on the variable level itself. Since the trend is more relevant than the level, these people are often labelled chartists, or trend chasers. This rule is often destabilizing, and it shows that agents forecast a similar (slightly smaller in absolute value) change in the variable from the change that they last observed:

$$Y^e = Y_{-1} + \phi \cdot (Y_{-1} - Y_{-2}) \quad 0 < \phi < 1 \quad (16N')$$

Finally, a more complex form of backward looking expectation is the so-called adaptive expectation rule. Under this rule, instead of using past income, agents use their own past error in predicting past income, to update their forecast. Since this more complex rule adds many previous periods as state variables (income in the past), we post-pone the study of this rule to the end of the paper.

Fundamentalist expectations

The same research mentioned above shows than next to the chartists or trend chasers, another rule that often becomes prevalent empirically is forward looking. In the simplest (more realistic) form, agents who are forward looking have an idea of the fundamental value (say the stationary state for

⁸ We add S to the equation number, to indicate we analyse stationary expectations. Other types of expectations will be discussed below and will have different letters: N – naïve; P –perfect foresight; F – fundamentalist.

instance), and they assume that the gap between last observation and the fundamental value will decrease proportionally each period. We saw an example of fundamentalist rule in the literature review above. It is clear that, while these agents believe that the system will eventually converge back to its proper, model consistent stationary state, they are much more realistic (and therefore empirically validated) than a simple stationary forecast, since this fundamentalist forecast is dynamic. When the system is beginning from the steady state, fundamentalists will also hold stationary expectations.

$$Y^e = Y_{-1} + \theta(Y^* - Y_{-1}) \quad 0 < \theta < 1 \quad (16F)$$

Agents holding this rule are also not rational, since they assume that income will move back to stationary state at a fixed, arbitrary rate, rather than through a dynamic path that is model consistent.

In case of enhanced fundamentalist expectations we find:

$$Y^e = \theta \cdot Y^* + (1 - \theta) \cdot [Y_{-1} + \Phi \cdot (Y_{-1} - Y_{-2})] \quad \Phi > 0 \quad (16F')$$

which includes 'simple' fundamentalist expectations (16F) for Φ equal to 0. We include this enhanced rule in our analysis, since this is a natural extension in line with our discussion of naïve expectations.

Adaptive expectations

There is evidence (Chiarella et al., 2014; Hommes, 2021) that agents form their beliefs according to some type of extrapolative rule. As mentioned above, some agents focus on the errors that they made in recent periods, and learn from them. This adaptive behaviour has the goal to gradually decrease forecasting errors. In the stationary state, this does indeed happen. If the economy is not moving, the expectations following this rule converge to the stationary state very quickly. When outside of the steady state, adaptive expectations (similarly to other backward looking rules) add persistence to the model.

$$Y^e = Y_{-1} + \theta(Y^e_{-1} - Y_{-1}) \quad 0 < \theta < 1 \quad (16A)$$

Adaptive expectations complicate our mathematical system, as these expectations take into consideration multiple preceding years. However this type of rule is also one of the most general versions of learning from past data applied in economics, after the well-known work on learning in (Evans & Honkapohja, 2001).

"perfect foresight"

Finally, as a useful comparison with models which do not describe boundedly rational expectations, we study the case of "perfect foresight". Under this rule, agents know the one model consistent expectation, given the shock, the form of the model *and* their own expectation, and they correctly forecast income (one period ahead) at every period in time:

$$Y^e = Y \quad (16P)$$

Summing up

Given the forms described in equations (16), it is clear that when the model is in the stationary state, all the expectation rules above become equivalent.

Finally, while these are the most widely used expectation rules, there are many other ways of forming expectations. We will elaborate on the various rules below and show their impact on the stability of

the economy. As G&L (p. 109) claim: “the model will easily accommodate any other scheme of expectations formation so long as it is not systematically perverse as would be the case if an error in one period were to be larger and in the same direction in the subsequent period.” However, they also show that in many cases the model then becomes too complicated to be solved analytically. We will also elaborate on this below. In section 3 we show how the model can be solved analytically for several types of expectations. We then introduce in section 4 more elaborate types of expectations in the model.

3 Solving the model

To further analyse the model, we look at its reduced form. Since we know that always should hold: $L = M + R = K$, the model excluding expectation formation can be reduced to the following four equations:

$$YD^e = (1 - \tau).Y^e - \delta.K_{-1} + r.\beta.Y_{-1} + r.R_{-1} \quad (18)$$

$$(1 + \mu - \tau - \beta).Y = \alpha_1.YD^e + \gamma.\kappa.Y^e + (\alpha_2 - 1 - r).\beta.Y_{-1} + (\alpha_2 - \gamma + \delta).K_{-1} + C^A + X^A + \alpha_2.R_{-1} \quad (19)$$

$$K = \gamma.(\kappa.Y^e - K_{-1}) + K_{-1} \quad (20)$$

$$R = X^A - \mu.Y + (1 + r).R_{-1} \quad (21)$$

Households accumulate wealth not only in the form of physical capital K and bonds issued by government B , but also in the form of foreign reserves R . However, for analytical reasons, we distinguish between two situations. First a ‘full open economy’, where the effects of foreign reserves on household income and wealth are included in the way described above. Second we analyse a ‘moderate open economy’ in which households in their consumption decisions ignore the foreign reserves that are indirectly accumulated and the corresponding interest income. This moderate open economy bears some similarities to the German and the Dutch situation.⁹

Combining equations (18) and (19), we can reduce the model to the following three equations:

$$\rho.Y = a_1.Y^e - a_2.Y_{-1} - a_3.K_{-1} + a_4.R_{-1} + C^A + X^A \quad (19')$$

$$K = \gamma.\kappa.Y^e + (1 - \gamma).K_{-1} \quad (20')$$

$$R = -\mu/\rho.a_1.Y^e - \mu/\rho.a_2.Y_{-1} - \mu/\rho.a_3.K_{-1} + (1 + r - \mu/\rho.a_4).R_{-1} + (1 - \mu/\rho).X^A - \mu/\rho.C^A \quad (21')$$

With:

$$\rho = (1 + \mu - \tau - \beta) > 0; a_1 = [\alpha_1.(1 - \tau) + \gamma.\kappa] > 0; a_2 = [1 + (1 - \alpha_1).r - \alpha_2].\beta > 0;$$

$$a_3 = [\gamma - \alpha_2 - (1 - \alpha_1).\delta] > 0; a_4 = (\alpha_1.r + \alpha_2) > 0.¹⁰$$

⁹ Both Germany and the Netherlands have a persistent current account surplus, increasing from 4 per cent of GDP on average prior to the financial crisis to 7 per cent of GDP thereafter. Net foreign debt fluctuates around 90 per cent of GDP in recent years. A typical Dutch feature is that, due to the funded pension system a lot of household savings end up in pension funds, who invest strongly abroad. The wealth of pension funds (over 250 per cent of GDP, and owned by households) is traditionally not included in household wealth in the Netherlands, when estimating the consumption function. See also Meijers and Muysken (2022).

¹⁰ In a moderate open economy we find $a_4 = 0$. The wealth effect α_2 , the depreciation rate δ and the interest rate r are assumed to be relatively small.

This is a dynamic system with output Y , capital K and reserves R as variables. We still need to specify the expectations.

We focus on the stationary state – i.e. output and capital do not grow over time. A stationary state solution then can be easily found if we assume expected output and capital to be equal to their stationary values. This solution is presented in section 3.1 for both the full open economy and the moderate open economy, and its properties are discussed. In section 3.2 we point out the self-fulfilling properties of optimism and pessimism in expectations in the model for a moderate open economy. We show in section 3.3 that this model is only stable when taxes and government debt are not too high.

3.1 The stationary state solution of the model

To analyse the stability properties of this system, we should first look at the stationary state solution. In stationary equilibrium all three variables Y , K and R , are constant over time, and expectations are consistent with the stationary state¹¹. We elaborate below that in case of a full open economy when the economy has a trade balance surplus in the stationary state ($\mu \cdot Y^* < X^A$), its foreign reserves have to be negative. The reason is that when reserves have to remain constant over time, the trade surplus has to be compensated by negative interest payments on foreign reserves. This observation provides a motivation to look also at the moderate open economy with stationary output. In that situation we assume that foreign reserves, which are indirectly accumulated by households, do not generate returns for households and therefore are also not recognised by households as part of their wealth ($a_4 = 0$). That is, households do only get returns on the deposits held at banks and on bonds, and these deposits and bonds are only included in the wealth accumulation affecting consumption. In that case positive foreign reserves can be consistent with a trade surplus. We elaborate both types of stationary equilibria below.

3.1.1 The full open economy

We assume that all three variables Y , K and R , are constant over time at Y^{**} , K^{**} and R^{**} , respectively, and expectations are consistent. We then find:¹²

$$Y^{**} = \frac{C^A + (1 - a_4 \frac{1}{r}) X^A}{\rho - a_1 + a_2 + a_3 \kappa - a_4 \frac{\mu}{r}} = \frac{C^A + (1 - \alpha_1 - \alpha_2 \frac{1}{r}) X^A}{(1 - \alpha_1) \cdot (1 - \tau - \delta \kappa + r \cdot \beta) - \alpha_2 \cdot (\kappa + \beta) + \mu (1 - \alpha_1 - \alpha_2 \frac{1}{r})} \quad (22)$$

$$K^{**} = \kappa \cdot Y^{**} \quad (23)$$

$$R^{**} = (\mu \cdot Y^{**} - X^A) / r \quad (24)$$

Equation (24) follows from the notion that in order to keep foreign reserves constant, imports should equal exports net of interest payments. This implies that when the economy has a trade balance surplus in the stationary state ($\mu \cdot Y^{**} < X^A$), the reserves have to be negative, i.e. the economy should have a foreign debt position. The reason is that when reserves have to remain constant over time, the trade surplus has to be compensated by interest payments on foreign debt (negative reserves).

According to equation (22) the impact of reserves has a negative impact on both the numerator and the denominator through the coefficient a_4 . The impact on the numerator is stronger, which implies that a higher value of a_4 leads to a lower value of stationary output Y^{**} . The reason is that reserves R^{**} are negative, while a higher value of a_4 indicates that the wealth effect of negative reserves is

¹¹ We indicate stationary state variables in the moderate open economy with a single *, and with a double ** in the full open economy.

¹² Since $a_4/r = \alpha_1 + \alpha_2/r > 1$ does probably hold, both the nominator and denominator of Y^{**} are negative.

stronger and/or that the interest payments on reserves play a more important negative role in consumption through their influence on disposable income. The other properties of the stationary state will be discussed in the next section, since they also apply to the moderate open economy.

3.2.2 The moderate open economy

In case of a moderate open economy output, capital and reserves are constant over time at Y^* , K^* and R^* , respectively, and expectations are consistent. We assume that foreign reserves, which are indirectly accumulated by households, do not generate returns for households and are also not recognised by households as part of their wealth in making consumption decisions ($a_4 = 0$). We find as stationary state solution of the model:¹³

$$Y^* = \frac{C^A + X^A}{\rho - a_1 + a_2 + a_3 \kappa} = \frac{C^A + X^A}{(1 - \alpha_1) \cdot (1 - \tau - \delta \kappa + r \beta) - \alpha_2 \cdot (\kappa + \beta) + \mu} \quad (25)$$

$$K^* = \kappa \cdot Y^* \quad (26)$$

$$R^* = (X^A - \mu \cdot Y^*) / r \quad (27)$$

Similar to G&L we assume that the denominator of (25) is positive, i.e. that does hold:¹⁴

$$1 - \alpha_1 > \frac{\alpha_2 \cdot (\kappa + \beta) + \mu}{1 - \tau - \delta \kappa + r \beta} \quad \text{and} \quad \tau < 1 - \delta \kappa + r \beta \quad (28)$$

in order to ensure $Y^* > 0$.¹⁵

The properties of the stationary state are discussed extensively in G&L for the situation without government. We therefore, focus here on the impact of introducing government in the model.

First, we observe that government expenditures are endogenous in the model due to the assumption of a limitation on government debt $\beta \cdot Y$ and endogenous taxes $\tau \cdot Y$. Combining (6), (7) and (8) yields the stationary state government expenditures to output ratio, $(G/Y)^*$:

$$(G/Y)^* = \tau - r \cdot \beta \quad (29)$$

As we explained above by choosing a higher debt-output ratio at a given tax rate, $(G/Y)^*$ decreases when β increases. Similarly, when the tax rate increases at a given debt-output ratio, $(G/Y)^*$ increases when τ increases. These observations imply that the impact of increased government expenditures does not follow from increased autonomous expenditures, equivalent to an increase in C^A in equation (19), but only indirectly through changes in β and τ .

A second observation is that the second condition in equation (28) puts a cap on the tax rate, which also puts a cap on government expenditures:

$$(G/Y)^* < 1 - \delta \cdot \kappa \quad (30)$$

However, within the admissible range, we observe that an increase in the tax rate τ always leads to an increase in stationary state output Y^* . This result is consistent with the reasoning behind our finding above that $(G/Y)^*$ increases when τ increases.

¹³ It can easily be verified, comparing equation (25) with (22) that stationary output in the moderate economy is higher than stationary output in the full open economy ($Y^* > Y^{**}$).

¹⁴ Equation (13) reduces to equation G&L equation (7.30) when we assume $\beta = \tau = \mu = 0$.

¹⁵ This is to ensure that 'the model makes sense.' (G&L, p. 229) See also G&L equation (7.31).

Finally, we observe from equation (25) that the impact of an increase in β on output depends on whether

$$\alpha_2 > (1 - \alpha_1).r \quad (31)$$

holds or not. If (31) does hold, the intuition is that the increase in consumption, due to the income effect of increased interest payments and the wealth effect of additional bonds, should be large enough to offset the increased interest payments by government, leading to an increase in government expenditures. However, the negative impact of β on $(G/Y)^*$ implies that condition (31) should not be taken for granted.

With respect to the foreign sector we find from equation (27) that an increase in exports X^A increases stationary output. An increase in the propensity to import μ leads to a decrease in stationary output. Both results are intuitively plausible.

Again we find that when the economy has a trade balance surplus in the stationary state ($\mu.Y^* < X^A$), the reserves have to be negative, i.e. the economy should have a foreign debt position. However, we show below that in the moderate open economy output and capital can remain at their stationary levels, while the economy has both a trade balance surplus and positive foreign reserves.

3.2 The impact of optimism and pessimism in expectations

For a better understanding of the role of expectations in our model we look at the impact of optimistic expectations. Let stationary output systematically be overestimated by Ω , that is:

$$Y^e = Y^* + \Omega \quad (16S')$$

whereby Y^* (or Y^{**}) is the stationary output consistent with these expectations. We can derive the impact of optimism for both types of open economies, and K^* (or K^{**}) the corresponding capital stock. As might be expected, the outcomes are different for the full open economy and the moderate one. We analyse first the implications for the moderate open economy and then those for the full open economy.

The model for the moderate open economy is given by equations (19') – (21'), with $a_4 = 0$. Substituting equation (16S') and $Y = Y_{-1} = Y^*$ and $K = K_{-1} = K^*$ yields as stationary state:

$$Y^* = \frac{C^A + X^A + \{\alpha_1.(1 - \tau) + [(1 - \gamma) + \alpha_2 + (1 - \delta).\alpha_1].\gamma.\kappa\}.\Omega}{(1 - \alpha_1).(1 - \tau - \delta.\kappa + r.\beta) - \alpha_2.(\kappa + \beta) + \mu} \quad (33)$$

$$K^* = \kappa.Y^* \quad (34)$$

From equation (33) we see that stationary output increases with Ω . In the case of optimistic expectations, overestimating output will result in higher stationary output due increased consumption and investment demand.¹⁶ This illustrates that optimism is a self-fulfilling prophecy in this model. And it is obvious that the same does hold for pessimism.

Similarly we find for the full open economy:

$$Y^{**} = \frac{C^A + (1 - \alpha_1 - \alpha_2 \frac{1}{r})X^A + [(1 - \gamma) + \alpha_2 + (1 - \delta).\alpha_1].\gamma.\kappa\}.\Omega}{(1 - \alpha_1).(1 - \tau - \delta.\kappa + r.\beta) - \alpha_2.(\kappa + \beta) + \mu(1 - \alpha_1 - \alpha_2 \frac{1}{r})} \quad (35)$$

¹⁶ If only consumers are optimistic, the term between brackets in the denominator of equation (18') would be equal to $\alpha_1.(1 - \tau)$. If only firms are optimistic, the term would be equal to $\gamma.\kappa$.

And the same holds as for the moderate open economy.

Finally, it is important to note that in this specification, where optimism or pessimism are introduced by adding Ω to expected output – cf. equation (16'S) – optimism does not affect the stability of the model (see below where stability is further analysed). Only the constant terms in equations (19') – (21') change, but this does not appear in the stability condition. That is, the impact of information from the past is not influenced by our specification of optimism or pessimism, and therefore there is no impact on the stability of the model.

Other forms of introducing optimism and pessimism might have an impact on stability, but our main point is that under plausible circumstances optimism and pessimism will have a tendency of self-fulfilling prophecies in the economy.

3.3 Stationary state stability conditions

We can derive the stability conditions for both type of open economies. As expected, these are different for the full open economy and the moderate one. The stability conditions for the full open economy can be derived, but they can only be analysed numerically. We therefore focus on the stability conditions for the moderate open economy, since these can be fully analysed.

3.3.1 Stability in the full open economy

In order to derive the stability conditions, we substitute the expectation equation (16S) in the model of equations (19') – (21'). We then find:

$$\rho.Y = -a2.Y_{-1} - a3.K_{-1} + a4.R_{-1} + C^A + X^A + a1.Y^{**} \quad (36)$$

$$K = (1 - \gamma).K_{-1} + \gamma.k.Y^{**} \quad (37)$$

$$R = -\mu/\rho.a2.Y_{-1} - \mu/\rho.a3.K_{-1} + (1 + r - \mu/\rho.a4).R_{-1} + (1 - \mu/\rho).X^A - \mu/\rho.C^A - \mu/\rho.a1.Y^{**} \quad (38)$$

Stability of the system then requires all eigenvalues of the coefficient matrix are within the unit circle.

After some manipulation, we derive that the three roots are given by:

$$x_1 = B - \left(\frac{X}{Z} + Z\right) \quad (39)$$

$$x_{2,3} = B - \frac{\left(\frac{X}{Z} + Z\right) \pm \sqrt{3} \cdot \left(\frac{X}{Z} - Z\right) \cdot i}{2} \quad (40)$$

As a consequence stability requires that:

$$\max \left[B - \left(\frac{X}{Z} + Z\right) ; \sqrt{\left[B - \frac{1}{2} \left(\frac{X}{Z} + Z\right) \right]^2 + \frac{3}{4} \cdot \left[\left(\frac{X}{Z} - Z\right) \right]^2} \right] < 1 \quad (41)$$

$$A = [a2.(2 - \gamma) - (1 - \gamma).p - a3.k + (1 - \gamma).a4.\mu]/(3.p)$$

$$B = [(2 - \gamma).p - a2 - a4.\mu]/(3.p)$$

$$C = [a2.(1 - \gamma) - a3.k]/(2.p)$$

$$X = A + B^2$$

$$Z = \{ [B^3 - C + (2/3).A.B]^2 - (A + B^2)^3 \}^{(1/2)} + B^3 - C + (2/3).A.B \}^{(1/3)}$$

Since this condition is very hard to analyse further, we have to resort to numerical simulations to investigate its properties. For the moment we leave this for further research.

However, if this condition is satisfied, the economy returns to Y^{**} , K^{**} and R^{**} after a shock. We illustrate this process for the case of a decrease in autonomous consumption. Initially, this leads to lower income: directly because consumption decreases, and indirectly through the multiplier-accelerator process. But lower income also leads to lower imports. Since exports remain at the same level, lower imports imply a decrease in foreign debt (increase in reserves). This leads to an increase in income through wealth and interest income. Both taxes and government expenditures will be lower because of the initial decrease in income – the latter since the amount of bonds will also decrease initially. However, in the end the total amount of bonds will return to its stationary level $\beta \cdot Y^{**}$. But total household wealth will decrease, since accumulated wealth has decreased due to lower savings during the transition period. On the other hand, accumulated foreign debt has decreased (reserves have increased) due to the lower imports during the transition period. Both household wealth and foreign reserves remain constant after the transition period. In the end foreign debt will be lower (reserves will be higher) at a level that compensates through wealth effects and interest income for the loss of income through lower autonomous consumption.

3.3.2 Stability in the moderate open economy

In order to derive the stability conditions for the moderate open economy, we substitute the expectation equation (16S) in the model of equations (19') – (21'), with $a_4 = 0$. That is, we assume that foreign reserves, which are indirectly accumulated by households, do not generate returns for households and therefore are also not recognised by households as part of their wealth. The implication is that the development of reserves over time does not affect output and capital accumulation. For that reason we can focus on the latter two variables. That is, we focus on the stability conditions for:

$$Y = - (a_2/\rho) \cdot Y_{-1} - (a_3/\rho) \cdot K_{-1} + [C^A + X^A + a_1 \cdot Y^*]/\rho \quad (42)$$

$$K = (1 - \gamma)K_{-1} + \gamma \cdot K \cdot Y^* \quad (43)$$

Stability of the model then requires that all eigenvalues of the coefficient matrix are within the unit circle, which implies:¹⁷

$$a_2/\rho < 1 \quad (44)$$

We note that in the stationary solution the capital stock is not affected by past output. That makes the dynamics of the model relatively simple. The stability conditions of other forms of expectations are much more complicated as we elaborate in section 4.

Substitution of a_2 and ρ in the stability condition (44) yields:

$$[(1 - \alpha_2) + (1 - \alpha_1) \cdot r] \cdot \frac{\beta}{(1 + \mu - \tau - \beta)} < 1 \quad (44')$$

A first observation is that this condition does always hold in absence of government. In that case the system is inherently stable, assuming the stationary expectations.

¹⁷ This is a sufficient condition, which should not be confused with the necessary condition that the absolute value of the system's determinant is smaller than unity, which is often used in G&L.

When we introduce government, we note that it is crucial whether $\beta + \tau - \mu$ is smaller or larger than unity. Now there are two reasons why in the stationary analysis we assume $\beta + \tau - \mu$ is smaller than unity, i.e. $\rho > 0$. First, when one considers the average EMU member state, the target value for β is 0.6, the average tax rate will be around 0.5 and the import quote will be around 0.5. Hence, only when β is considerably larger than 0.6 this condition is violated. However, one should realise that the target value of β is 0.6, is based on a growing economy. In our stationary economy this target value does not hold. If positive at all, it will be much lower – this is the second reason why we assume $\beta + \tau - \mu$ is smaller than unity.

As we mentioned above, the stationary state government expenditures to output ratio is given by equation (29). This equation implies that for positive values of β , the stationary state requires a budget surplus:

$$\tau - (G/Y)^* = r.\beta \quad (29')$$

This surplus is necessary to pay for the interest on government debt implied by the positive value of β .

Since $\tau < 1 + \mu - \beta$, we find from (35') as the stability condition:

$$\tau < 1 + \mu - \beta - [(1 - \alpha_2) + (1 - \alpha_1).r].\beta \quad (45)$$

Since $[(1 - \alpha_2) + (1 - \alpha_1).r].\beta > 0$, this condition can be binding.

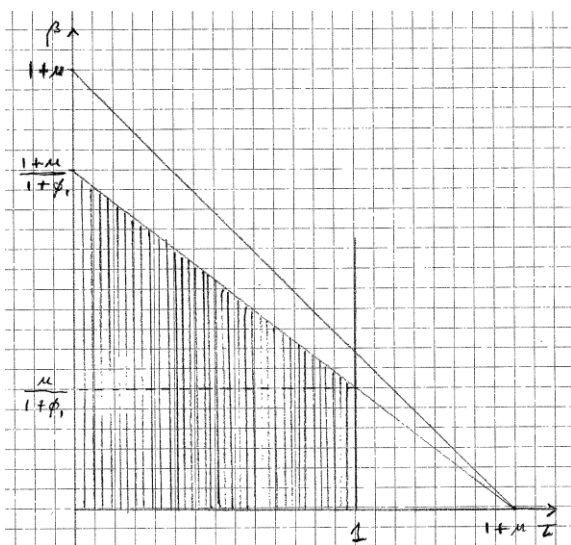
To illustrate this condition, rewrite the borderline case as:

$$\beta = \frac{1 + \mu}{1 + \phi_1} - \frac{\tau}{1 + \phi_1} \quad (46)$$

with $\phi_1 = [(1 - \alpha_2) + (1 - \alpha_1)r] > 0$.

This borderline case is displayed in Figure 1,¹⁸ and the region of stability is displayed by the vertically striped area.

Figure 1 Stability conditions for stationary expectations



¹⁸ Here $\tau = 0$ gives: $\beta = \frac{1 + \mu}{1 + \phi_1}$, and $\tau = 1$ gives: $\beta = \frac{\mu}{1 + \phi_1}$. Remember from equation (27) that also should hold: $\tau < 1 - \delta.\kappa + r.\beta$, we ignore this in the figure.

In Figure 1 we present at the vertical axis the debt-to-output ratio β , and on the horizontal axis the tax rate τ . The region of stability is bounded by a downward sloping line, which implies that at a higher debt to output ratio the admissible tax rate to maintain stability is lower. The intuition for this trade-off follows from the binding constraint on government expenditures by the debt-output ratio. This implies a trade-off between the tax rate and debt-output ratio in the multiplier impact of government expenditures (in addition to the impact of the tax rate through disposable income).

Also, the admissible tax rate at a certain debt to output ratio is even lower when μ decreases. This follows from the observation that a lower propensity to imports increases the multiplier and hence the impact of shocks to the economy.

Moreover, from the properties of ϕ_1 we infer additional properties of the model, which are intuitively plausible. One observes that when adjustment of the capital stock becomes faster, i.e. γ increases, the model becomes more stable since it adjusts faster to the new situation. The same does hold for the wealth effect in consumption – when α_2 increases, the model becomes more stable. This finding is consistent with G&L (p.83).

In summary, we find that under stationary expectations for the moderate open economy the probability of stability increases when: (1) the tax rate is lower; (2) the debt-to-output ratio decreases; (3) the propensity to imports is higher; (4) the adjustment of the capital stock is faster; (5) the wealth effect on consumption is higher; (6) the propensity to consume is higher; and (7) the interest rate is lower.

A final observation is that stability only refers to output and capital. After a shock the economy returns to Y^* and K^* . However, it does not return to R^* . Take for example situation with initially a positive trade balance and corresponding negative reserves in the stationary state. Then we follow the example from the previous section, with a decrease in autonomous consumption. This leads to lower income: directly because consumption decreases, and indirectly through the multiplier-accelerator process. Moreover, the decrease is stronger compared to the full open economy, because there is no countervailing tendency from returns to reserves, and wealth effects. Eventually income and capital will return to their new stationary state levels.

However, reserves will increase consistently and become positive. When Y^* has reached its new level, reserves will continue to increase according to:

$$R = X^A - \mu \cdot Y^* + (1 + r) \cdot R_{-1} \quad (47)$$

As we explain in Appendix A, this does not affect output and the capital stock, since it will be absorbed by the financial sector. This is consistent with the German and Dutch situation, as we explained above.

4 The role of expectations in the stability of the model

To illustrate the importance of the nature of expectations we analyse now the impact of the various types of expectations introduced in section 2.3 on the stability of the model. We focus on the case of a moderate open economy since this can be fully analysed, as we elaborated in the previous section. Consequently, our attention is directed towards the evolution of output and capital stock.¹⁹

The border lines have been derived using the analytically derived stability conditions, see Appendix B1, except for adaptive expectations. Unfortunately the stability conditions are too complex to analyse them in a straight forward way, as we did in section 3.3.2. For that reason we investigate the stability conditions numerically, using plausible parameter values – see Meijers and Muysken (2022). These parameter values are reproduced in Table 3.

Table 3. Parameter settings for the stationary expectations model simulations

Imputed values			
$\alpha 1$	0.7	μ	0.5
$\alpha 2$	0.02	β	0.4
γ	0.5	τ	0.4
δ	0.08	X^a	100
κ	1	C^a	25
r	0.01	θ	0.5
Calculated values			
ρ	0.7	a2	0.3932
a1	0.92	a3	0.456

When we substitute the expectations equations (16..) in (19') and (20'), the generic reduced form of the moderate economy model is defined by:²⁰

$$Y = a.Y_{-1} + b.K_{-1} + e \quad (48)$$

$$K = c.Y_{-1} + d.K_{-1} + f \quad (49)$$

and various forms of expectations define the parameters of that model. These parameters are summarized in Table 4.

Table 4. Coefficients a, b, c and d for various type of expectations

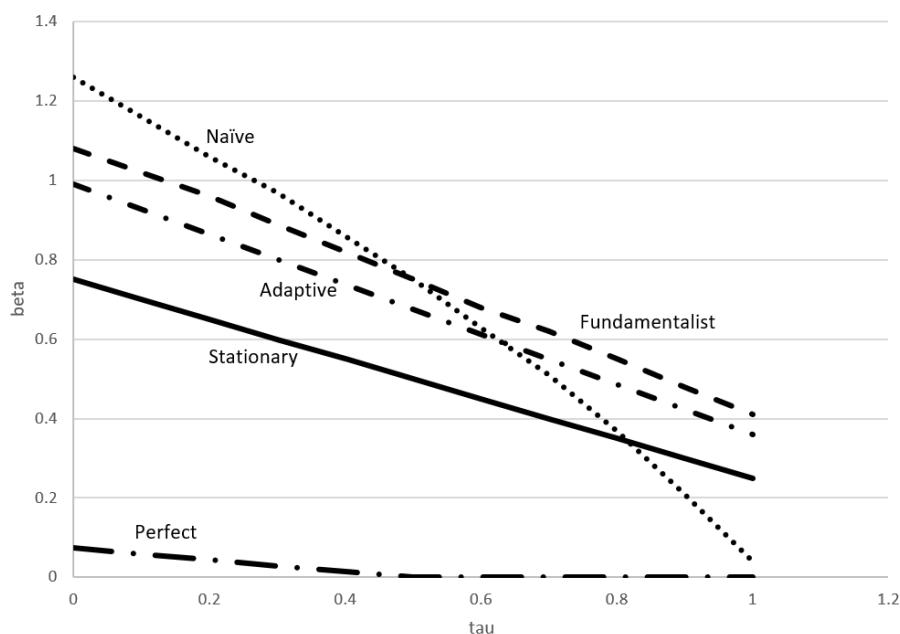
	a	b	c	d
Stationary	$-(a2/\rho)$	$-(a3/\rho)$	0	$(1 - \gamma)$
“perfect foresight”	$-a2/(\rho - a1)$	$-a3/(\rho - a1)$	$-a2.\gamma.\kappa/(\rho - a1)$	$(1 - \gamma)$ $-a3.\gamma.\kappa/(\rho - a1)$
Naïve	$(a1 - a2)/\rho$	$-(a3/\rho)$	$\gamma.\kappa$	$(1 - \gamma)$
Fundamentalist	$[a1.(1 - \theta) - a2] / \rho$	$-(a3/\rho)$	$(1 - \theta).\gamma.\kappa$	$(1 - \gamma)$

¹⁹ Note that equation (47) indeed shows that the development of foreign reserves is not stationary in the moderate open economy if we assume a neutral trade balance, but given the fact that we assume a moderate open economy, there is no feedback from foreign reserves to the remainder of the model.

²⁰ In case of enhanced naïve and fundamentalist expectations, see equations (16N') and (16F'), the model becomes more complex. This is presented in Appendix B2.

For each of the expectations forms presented in Table 4 and for adaptive expectations, we then check the values of τ and β , for which the model is stable, conditional on all other parameters as presented in Table 3.²¹ This enables us to derive the border lines of stability analogous to the border line presented in Figure 1 for the stationary state. The findings are summarised in Figure 2.

Figure 2 Stability conditions for various type of expectations



We focus on the values of τ and β , since these are policy parameters which are set by fiscal policy. Analysing the space of stability in this setting provides an interesting and policy relevant context to illustrate the impact of the nature of expectations on the stability of the model. An additional reason is that both τ and β have a substantial influence on the stability of the model. This is not the case for the interest rate r , which is affected by monetary policy. Therefore we focus on the fiscal policy parameters in our model.

We take stationary expectations, which we analysed in section 3.2.1 as the benchmark. The stability condition (35') is reproduced in Figure 2, and we add the conditions for “perfect foresight”, naïve expectations, fundamentalist expectations and adaptive expectations.

One observes that the conditions under “perfect foresight” are more binding (actually saddle point stable), while those for naïve expectations are less binding than stationary expectations. This is not consistent with the findings in the literature as reviewed in section 2.3.

The reason for our findings lies in the demand oriented nature of our model, where a positive shock in output leads to higher output due to the multiplier-accelerator mechanism.

When comparing a positive shock in output under “perfect foresight” to stationary expectations, the shock in output will directly translate in enhancing output under “perfect foresight”, while it will not do so under stationary expectations. Hence the initial impact under “perfect foresight” is larger and will therefore destabilise the model stronger.

²¹ Numerical simulations are not as precise as analytical solutions which implies that borderline solutions for the analytical model might not be applicable for the numerical simulation model. This also implies that numerical simulation is not suitable to find the specific or precise stability conditions in the case of adaptive expectations.

On the other hand, under naïve expectations, the shock in output will directly affect lagged output, whereas it will affect output one period later under stationary expectations. Therefore the feedback is stronger under naïve expectations and this explains why the model is more stable.

We look at the impact of expectations more in detail below.

4.1 Saddle point stability under “perfect foresight”

With “perfect foresight” (16P) the reduced form of the relevant equations (19’) and (20’) becomes:

$$Y = -a2/(\rho - a1).Y_{-1} - a3/(\rho - a1).K_{-1} + (C^A + X^A)/(\rho - a1) \quad (50)$$

$$K = \gamma.k.Y + (1 - \gamma).K_{-1} \quad (51)$$

An important observation is that shocks in output did not affect the capital stock through past output under stationary expectations. Under “perfect foresight” these shocks have an instantaneous impact on capital and through lagged capital also on output. This implies that a shock in for instance exports X^A has a direct positive impact on both output and capital through current output, and then also indirectly an additional positive impact through both past output and capital.

This explains why the line of stability under “perfect foresight”, represented by the bottom line in Figure 2, is not the upper bound of the border of stability, but represents those combinations of tax rate and debt ratio for which the system is saddle point stable.²²

Actually, this is a surprising result, since in most models perfect foresight achieves the widest parameter space where the model is stable. It is easy to see why: when agents have the complete information, they do not waste time and resources into arbitrary errors, but right away re-optimize their decisions, which puts them on the equilibrium path that leads to the stable stationary state (Hommes, 2021; Kurz et al., 2013).

4.2 Less instability under naïve expectations

With simple naïve expectations (16N) the reduced forms of the relevant equations (19’) and (20’) become:

$$Y = (a1 - a2)/\rho.Y_{-1} - a3/\rho.K_{-1} + (C^A + X^A)/\rho \quad (52)$$

$$K = \gamma.k.Y_{-1} + (1 - \gamma).K_{-1} \quad (53)$$

An important difference with stationary expectations is that the capital stock is influenced by past output. This deflects the destabilising impact of current output through the capital stock and creates an additional dynamic loop compared to stationary expectations. That is, past output does not only influence current output directly, but also indirectly through the lagged capital stock. Moreover, past output influences current output with coefficient $(a1 - a2)/\rho$, compared to $-a2/\rho$ under stationary expectations. In that respect the multiplier accelerator mechanism through $a1$ plays a stabilising role.²³ In addition, lagged income appears in the system through the interest payments and wealth effects related to government debt. If government debt increases this can lead to a destabilising impact of government debt – this is obvious in the case of stationary expectations as can be seen from Figure 1.²⁴ In the case of naïve expectations the additional dynamic loop through the capital stock provides

²² For all combinations of τ and β not on this line we find that the stability conditions are not satisfied numerically.

²³ The coefficient $a1 = [\alpha1.(1 - \tau) + \gamma.k]$ contains both the multiplier and the accelerator.

²⁴ This effect is found in the coefficient $a2 = [1 + (1 - \alpha1).r - \alpha2].\beta$.

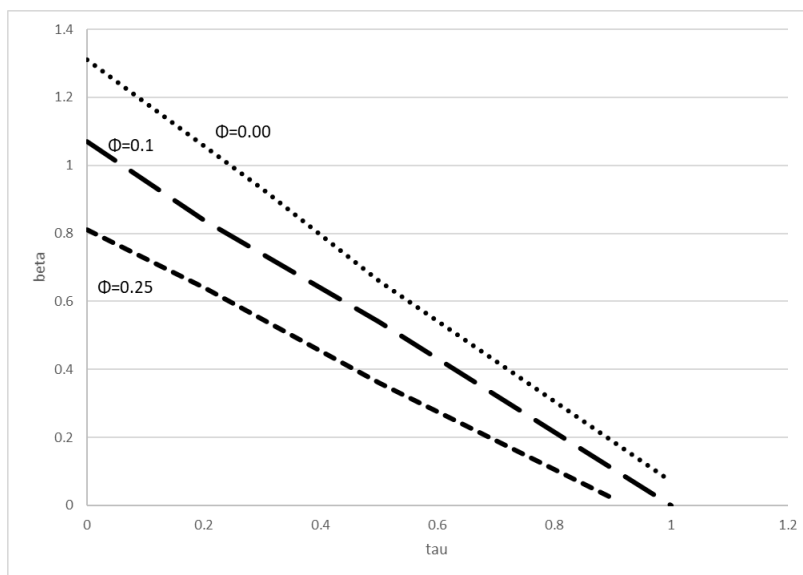
a counterbalancing effect, since the influence of lagged capital on output is negative because of the lagged adjustment of the capital stock to its target value.²⁵ This enhances the stability of the model.

The borderline for naïve expectations is presented by the dotted line in Figure 2. One observes that under the chosen parameter conditions the model is more stable than under stationary expectations till a high tax rate. The reason for the latter is that for an increasing tax rate the influence of the lagged capital stock becomes increasingly stronger relative to the influence of lagged output, since the multiplier effect decreases. At some level of the tax rate the negative relative influence of the lagged capital stock then is so dominating that it becomes destabilising.

Hence, under simple naïve expectations the model becomes more stable – there are less restrictions necessary to guarantee stability of the model. The intuition is that the path dependency of naïve expectations makes the reaction to shocks in the economy smoother.

When we assume enhanced naïve expectations (16N') the stability conditions change (Appendix B2). The reason is that past changes in output also influence expectations, and after a shock past changes have changed. It is plausible that the stronger the impact of past changes is, the less stable the model becomes. This can also be observed from Figure 3 where the weights of past changes in output on expectations increase from $\phi = 0$ (naïve expectations as above) to $\phi = 0.25$, thereby lowering the stability corridor.

Figure 3. Stability conditions for enhanced naïve expectations, different weights of past changes in output



4.3 Intermediate instability under fundamentalist expectations

With simple fundamentalist expectations (16F) the reduced forms of the relevant equations (19') and (20') become:

$$Y = [a_1 \cdot (1 - \theta) - a_2] / \rho \cdot Y_{-1} - a_3 / \rho \cdot K_{-1} + (C^A + X^A + a_1 \cdot \theta \cdot Y^*) / \rho \quad (54)$$

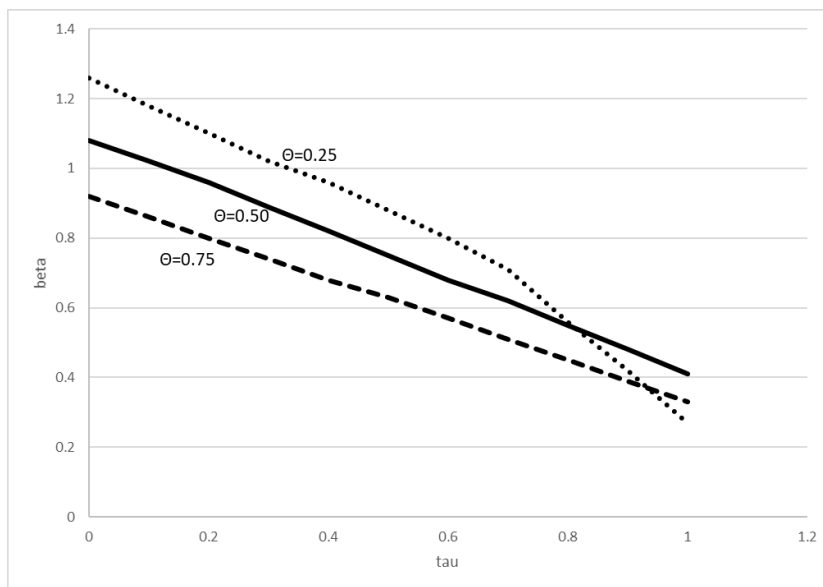
$$K = (1 - \theta) \cdot \gamma \cdot K \cdot Y_{-1} + (1 - \gamma) \cdot K_{-1} + \gamma \cdot K \cdot \theta \cdot Y^* \quad (55)$$

²⁵ This follows from our assumption that $a_3 = [\gamma - \alpha_2 - (1 - \alpha_1) \cdot \delta] > 0$, which implies that we assume a positive speed of adjustment of the capital stock, since should hold $\gamma > \alpha_2 + (1 - \alpha_1) \cdot \delta > 0$.

The borderline for fundamentalist expectations is presented by the solid line in Figure 2 (for $\theta = 0.5$). Since fundamentalist expectations are a weighted average of stationary and naïve expectations, one would think that the model is more stable than under stationary expectations, but less stable than under naïve expectations. This does hold indeed for lower and intermediate values of the tax rate. However, for higher values of the tax rate fundamentalist expectations become less stable because at a very high tax rate the multiplier becomes ineffective, as we explained for naïve expectations above.

It seems plausible that for a higher speed of adjustment of expectations, the region of stability decreases, in line with our intuition above that a larger degree of path dependency makes the adjustments to shocks in the economy smoother and hence increases stability. This intuition is confirmed by Figure 4, which shows that the region of stability decreases when the speed of adjustment increases. For higher values of the tax rate, fundamental expectations become progressively less stable, at a low speed of adjustment. The first reason is that as we discussed in section 3.1.2 steady state output Y^* increases with the tax rate, but this effect is less important at a low speed of adjustment.²⁶ Next to that, as we saw in Figure 2, for higher values of the tax rate, naïve expectations become less stable because at a very high tax rate the multiplier becomes ineffective, at a low speed of adjustment.²⁷

Figure 4 Stability conditions for fundamentalist expectations, different speeds of adjustment

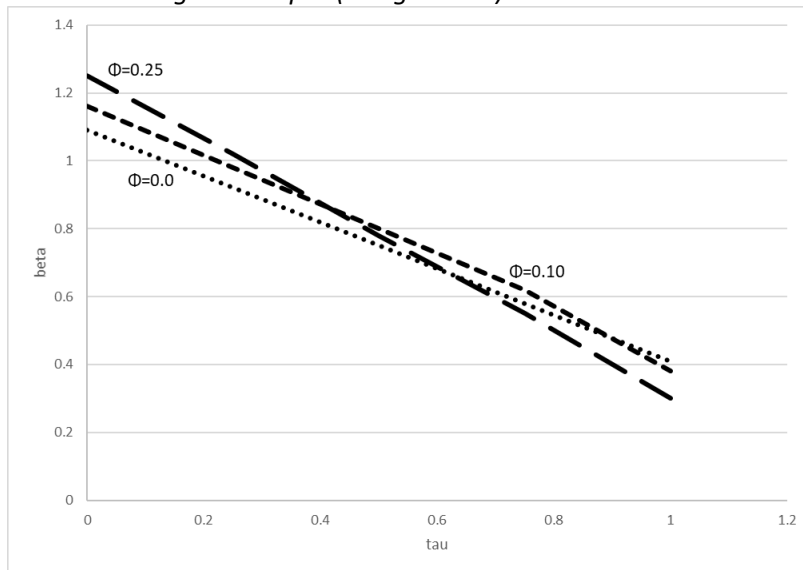


When we assume enhanced fundamentalist expectations ($16F'$) the stability conditions change (Appendix B2) – this is in line with our discussion of naïve enhanced expectations. Again we would expect that the stronger the impact of past changes is, the less stable the model becomes, but this will be moderated by the impact of stationary expectations on fundamentalist expectations. This can also be observed from Figure 5.

²⁶ The lower the speed of adjustment θ is, the less important Y^* is in expectations formation.

²⁷ Although tempting, it is not correct to use the results of a linear combination of Figures 2 and 3 to compare these with Figure 5. Appendix B shows that the stability borders are not linear combinations of β and τ .

Figure 5. *Stability conditions for enhanced fundamentalist expectations, different weights of past changes in output (using $\theta = 0.5$)*



At a relatively low impact of past changes ($\phi = 0.10$), the changes in output only start to dominate at a very high value of the tax rate. At a higher impact, past changes start to dominate at a lower value of the tax rate.

4.4 Varying instability under adaptive expectations

As we mentioned above, the stability conditions for adaptive expectations cannot be solved analytically. That is, we cannot present the reduced form of the relevant equations (19') and (20') with adaptive expectations. The reason is that adaptive expectations include a second dynamic element in the model such that investigating stability requirements include higher-order calculations. Under adaptive expectations, we have for expected output:

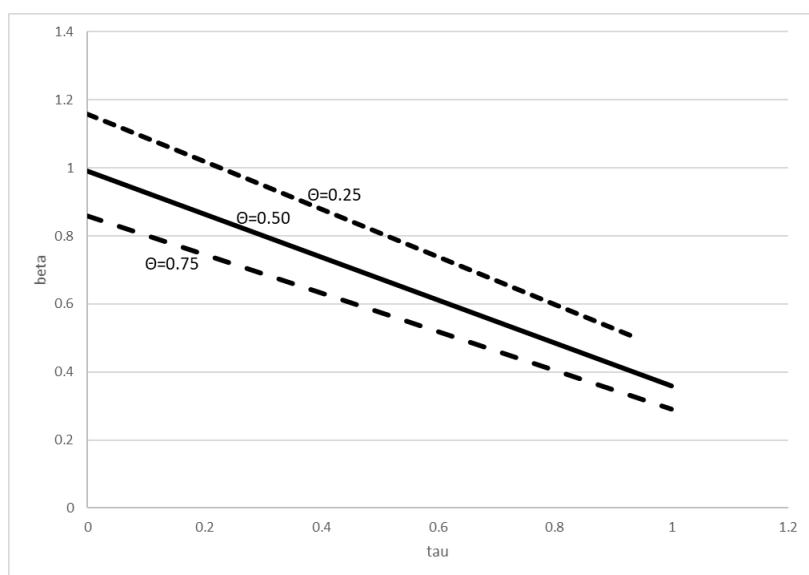
$$y_t^e = \theta y_{t-1}^e + (1 - \theta) \cdot y_{t-1} \stackrel{\text{def}}{=} y_{t-1} + \theta(y_{t-1}^e - y_{t-1}) \quad (16A)$$

And substituting y_t^e backwards shows that y_t^e itself depends on the entire history of y_t .

Given the structure of the model, the stability condition of the model cannot be found analytically and we therefore continue by numerical simulations of the whole model.

We start in the stationary state and simulate the model numerically, using the Eviews software package, where the expected output is equal to output in the stationary state. The model then remains in a stationary state. We shock the model in a single period by increasing the expected output by 10%. The model then either returns to a stationary state after the shock, in which case the model is stable or does not return to a stationary state, in which case the model is unstable. As before, we check the values of τ and β , for which the model is stable or unstable, conditional on all other parameters as presented in Table 3. The resulting values of τ and β constitute the border of stability represented by the dash-dotted line in Figure 2.

Figure 6 Stability conditions for adaptive expectations, different speeds of adjustment



As we observe from the figure, the region of stability for adaptive expectations is below that of fundamentalist expectations, implying that the region of stability is somewhat smaller. However, it is above the region of stability for stationary expectations. That is, expected income adjusts under adaptive expectations between ultimately stationary income and lagged (naïve expected) income.

Finally, as in the case of fundamentalist expectations, we find that for a higher speed of adjustment the region of stability decreases - see also Figure 6. This is in line with our intuition above that a larger degree of path dependency smoothens the adjustments to shocks in the economy and hence increases stability. Again, for higher values of the tax rate naïve expectations become less stable because at a very high tax rate the multiplier becomes ineffective, at a low speed of adjustment.

4.5 Stability under various expectations compared

Section 4 shows that the stability of the SFC model under different forms of expectations varies greatly. What is more surprising, the qualitative results arising from macroeconomic models with optimising agents do not hold in SFC models. We now study the natural differences between the two approaches, in order to understand the roots of this qualitative result.

When we compare the two models, the DSGE approach follows a (linear or not-linear) expansion around an existing and often globally stable steady state. Therefore in the absence of frictions, this setup will mechanically lead to its stationary equilibrium. This feature of DSGEs is a double edge sword, since on the one hand it makes the model tractable and it builds-in a version of the “invisible hand”, through an economy which mean-reverts towards steady state. On the other hand this literature has struggled to match the persistence and endogenous volatility typical of financial crises (Wieland & Wolters, 2011).

Looking at SFC models, these models are not built to be anchored at a steady state. Agents linearly extrapolate from the current conditions with simple backward looking rules. In this set-up, the lack of frictions can actually accelerate the endogenous (even potentially explosive) dynamics, instead of leading it back to a stable equilibrium. Agents in SFC models use thumb rules to take their decisions, instead of re-optimising their actions when they change expectations. Therefore the result that naïve expectations have a wider policy space of stability is due to the fact that as long as agents do not consciously re-optimize their decisions, updating their expectations in real time and aligning them

with the “perfect foresight” rule is actually leading the model to stronger reactions to exogenous shocks, and therefore to less stability. On the contrary, naïve expectations, which take a full lag before reacting to shocks, are among the most stable rules, due to this delay.

This result aligns with recent interdisciplinary literature showing that in a complex world which contains fundamental uncertainty and where optimization is not feasible, it is simple thumb rules (also called heuristics, meaning the easy forms listed above, like naïve, fundamentalist, adaptive) that are the closest one can do to get to a better result. In this world without optimisation, perfect foresight is not the first best, but simple thumb rules are (Dosi et al., 2020; Gerd Gigerenzer, n.d.; Gigerenzer & Brighton, 2009). In our paper we show that this intuition is easily carried over into SFC models, given these models' more realistic description of decision making processes. Ultimately, this section shows that the space of feasible rules for fiscal policy (in terms of tax rate and debt level) is larger when heuristics and thumb rules are the norm than it is for “perfect foresight”.

5 The role of expectations in the adjustment to shocks

To further examine the influence of different expectation regimes on the dynamic behaviour of the model, we proceed in this section by presenting results obtained from numerical simulations. We follow the adjustment in output and capital to shocks, while returning to a stable equilibrium.

The focus remains on the moderate open economy. The model is exposed to two different impulses, one related to expected output and the other is related to exports. An impulse to the expected output will directly influence expectations, and also the subsequent development of these expectations, depending on the expectations regime. Moreover, this impulse will directly affect both output and capital formation as follows from equations (19') and (20'). In this case, both consumers and firms are subject to the same impulse in their expectations. An impulse to exports will impact output only, and via that route expected output. Consumers will be affected differently than firms. As a result, the dynamics of the responses will be different. In each scenario, a single positive impulse of 10% is applied.

We first discuss the behaviour of the model in case of simple expectations – cf. the expectations compared in Figure 3 – in section 5.1. Then we analyse the case of enhanced expectations in section 5.2. Some concluding remarks are made in section 5.3.

5.1 Simple expectations

We start with a discussion on the behaviour of the model with simple expectations – that is, the expectations presented in Figure 3, except perfect foresight.²⁸

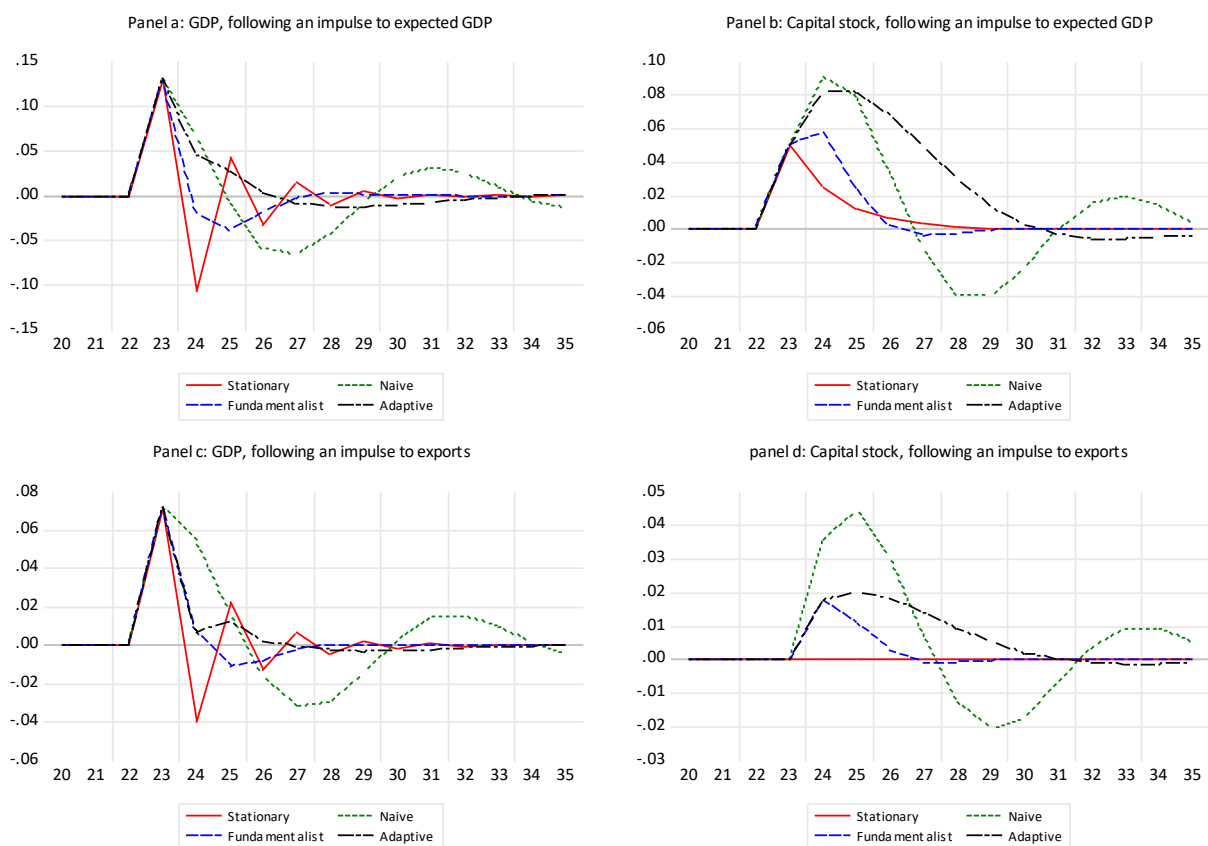
The evolution of GDP following the impulse on expected GDP is presented in Figure 7, Panel a, while Panel b illustrates the evolution of the capital stock. Initially, the reactions are the same across all expectation regimes, but they diverge thereafter. Regardless of the expectation regime, the responses after a shock to output converge towards the stationary state. However, the oscillations are more pronounced in the short run for the stationary expectations model, whereas under naïve expectations, the oscillations persist over a longer period. It's worth noting that the initial deviation exceeds 10% since the impulse affects both consumption and investments immediately through its impact on expected output.

²⁸ Since perfect foresight is saddle-point stable, numerical simulations are not possible.

Since investments do not instantly bridge the gap between the target capital and the current stock ($\gamma = 0.5 < 1$), the capital stock responds by 5% after the first year following the impulse in expected output. As anticipated, in the case of stationary expectations, the capital stock immediately adjusts towards the stationary state. Under the fundamentalist expectations, the capital stock increases slightly further and also converges rapidly to the stationary state. Conversely, for naïve and adaptive expectations, the capital stock initially continues to increase and gradually moves back to the stationary state, again exhibiting pronounced oscillations in the case of naïve expectations.²⁹

When an impulse is applied to exports, it initially affects output following equation (19'). However, investments are only influenced by changes in expectations and these expectations are only delayed affected by the impulse. Consequently, the response in output is approximately 7%, which is about half in size compared to an impulse on expected output, as depicted in Panel c of Figure 7.

Figure 7. Relative impulse responses of the moderate open economy under various expectations regimes



The movement of output is similar to the impulse on expected output, with adaptive expectations showing a slightly faster adjustment. However, the behaviour of the capital stock differs. As shown in Panel d, stationary expectations have no impact on the capital stock. In the case of fundamentalist expectations, the capital stock quickly reverts back to the stationary state. Conversely, naïve expectations exhibit substantial and prolonged oscillations once again.

Table 5 provides a comprehensive analysis of the size effects of different expectation regimes following impulses to the economy. It presents the sum of absolute values of relative deviations for both GDP and the capital stock after applying identical impulses to either expected output or exports

²⁹ As we already mentioned in the previous section n. 27, it is not correct to use the results of a linear combination of expectations in Table 5 to compare these with Table 6.

over a period of 30 years, which seems long enough to capture even slow convergence to the stationary state.

Table 5. Cumulative relative deviations from stationary state values under various expectations regimes

		$\beta = 0.4, \tau = 0.4$			
		GDP		Capital stock	
Regime	Impulse	Y^e	X	Y^e	X
	Stationary		35%	16%	10%
Naïve		54%	31%	47%	23%
Fundamentalist		22%	10%	14%	3%
Adaptive		27%	11%	41%	9%

The findings reveal that the impact of impulses on expected output has approximately twice the magnitude compared to impulses on exports, regardless of the expectation regime. This holds true across all expectation regimes examined. Specifically, the cumulative deviations of GDP are significantly larger when impulses are applied to expected output.

However, when considering the cumulative deviations of the capital stock, distinct patterns emerge. For stationary and fundamentalist expectations, the deviations are either zero or very small, respectively. Firms adjust the capital stock to stationary or near stationary values, so leading to zero or minimal deviations from stationary state. In contrast, adaptive expectations result in slightly larger cumulative deviations, while naïve expectations yield substantial deviations, indicating pronounced oscillations in the capital stock under naïve expectations.

We repeat the simulation with different values for β and for different values for τ to investigate the effect of various values of the size of government debt and taxes. The cumulative deviation in case β is increased from 0.4 to 0.5 is presented in the left four columns in Table 6 and the cumulative deviation in case τ is increased from 0.4 to 0.5 is presented the right four columns of that table.³⁰

Table 6. Cumulative relative deviations from stationary state values under various expectations regimes using different values for government debt and taxes.

		$\beta = 0.5, \tau = 0.4$				$\beta = 0.4, \tau = 0.5$			
		GDP		Capital stock		GDP		Capital stock	
Regime	Impulse	Y^e	X	Y^e	X	Y^e	X	Y^e	X
	Stationary		98%	46%	10%	0%	48%	23%	10%
Naïve		72%	40%	58%	27%	72%	43%	59%	30%
Fundamentalist		26%	11%	14%	3%	25%	11%	14%	4%
Adaptive		29%	12%	41%	9%	29%	12%	41%	10%

An increase in the debt-to-output ratio from 40 to 50% leads to large deviations in GDP in the responses to an impulse in expected output under stationary and naïve expectations. Especially under stationary expectations the oscillations that are presented in Figure 7 are increasing for increased

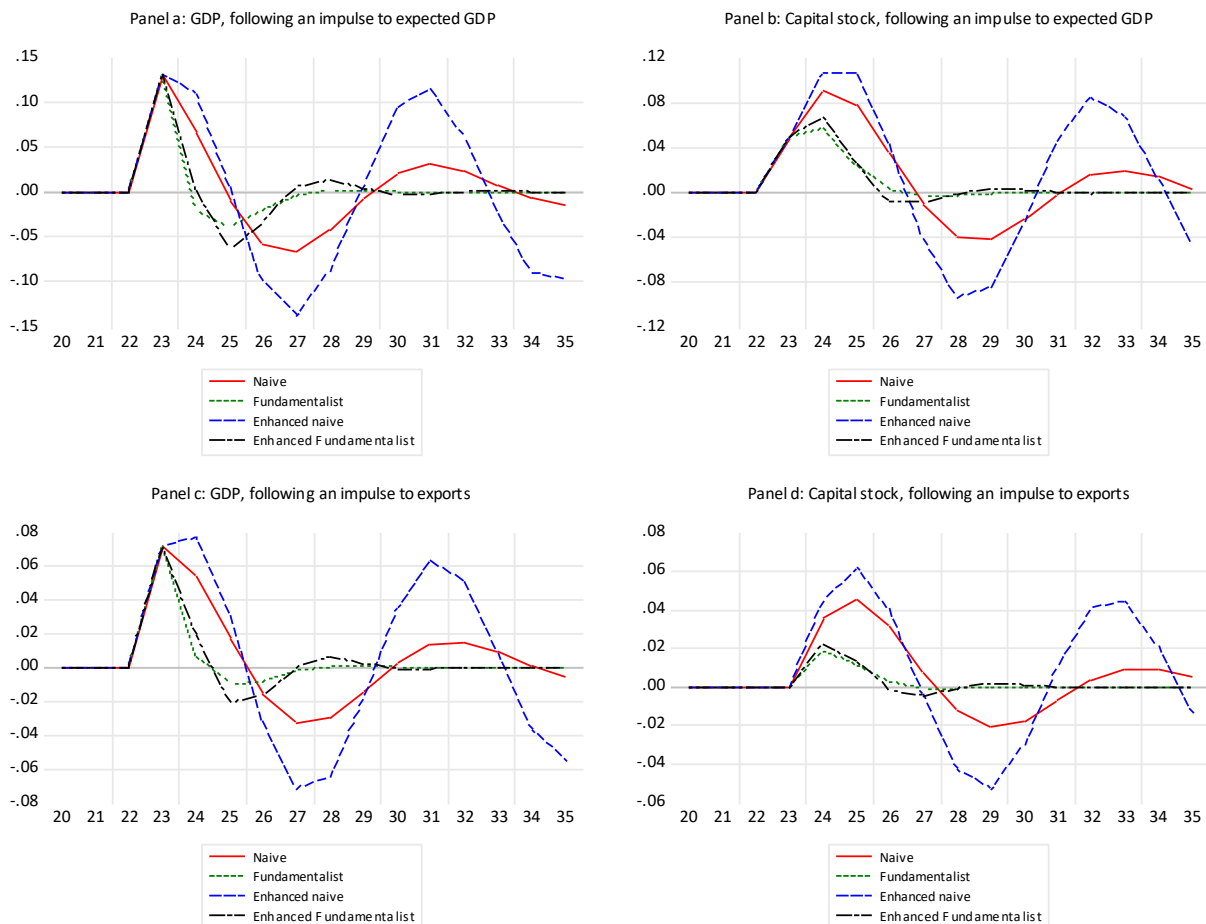
³⁰ A combination of $\beta = 0.6$ and $\tau = 0.4$ leads to an unstable model in case of stationary expectations, see Figure 2. Therefore, using $\beta = 0.5$ is more appropriate

values of the debt ratio, implying larger cumulative deviations following an impulse. Indeed, a combination of $\beta = 0.5$ and $\tau = 0.4$ shows to be close to the border of the stability corridor under stationary expectations as presented in Figure 2. In case of stationary expectations, the responses of the capital stock are smaller and even zero in case of an impulse to expected exports. If firms target their investments on stationary output, there is no effect of changes in expected exports to the capital stock, cf. equation (43). In all other cases the responses under increase tax rate are the same or very comparable to the responses under an increased debt rate. Fundamentalist expectations tend to result in the least cumulative deviations, indicating that the combined speed and amplitude of adjustments are minimized.

5.2 Enhanced naïve and fundamentalist expectations

Including longer lags in expectations formation leads to smaller stability corridors for government policy. As is depicted in Figure 3 above, under enhanced naïve expectations the area of stability becomes smaller for Φ equal to 0.25. Figure 8 illustrates the impact of an impulse under enhanced naïve expectations and enhanced fundamentalist expectations, using Φ equal to 0.25 and compares the responses with simple naïve and simple fundamentalist expectations in which Φ is equal to 0.³¹

Figure 8 Enhanced naïve and enhanced fundamentalist expectations.



Changes under fundamentalist expectations are only slightly different because fundamental values remain unchanged under enhanced expectations. This is different under naïve expectations as lagged realized values are entirely driving expectations and increasing Φ will only amplify the cycles after an

³¹ In all cases presented in this section, Θ is equal to 0.5

impulse to expected GDP and to expected exports. For even larger values of Φ oscillations increase over time and the model is unstable.

Table 7. Cumulative deviations relative to stationary state values for simple and enhanced expectations regimes for different values for government debt and taxes

Regime	$\beta = 0.4, \tau = 0.4$				$\beta = 0.5, \tau = 0.4$				$\beta = 0.4, \tau = 0.5$			
	GDP		Capital stock		GDP		Capital stock		GDP		Capital stock	
	Y^e	X	Y^e	X	Y^e	X	Y^e	X	Y^e	X	Y^e	X
Naïve	54%	31%	47%	23%	72%	40%	58%	27%	72%	43%	59%	30%
Enhanced naïve	126%	72%	99%	51%	262%	147%	187%	99%	236%	136%	168%	93%
Fundamentalist	22%	10%	14%	3%	29%	12%	41%	9%	29%	12%	41%	10%
Enhanced fundamentalist	25%	13%	16%	4%	32%	15%	17%	5%	29%	15%	17%	5%

In Table 7 we present the deviations of GDP and the capital stock from stationary state after an impulse to expected GDP and to expected exports under various values of the debt-to-output ratio and the tax rate, all for Φ being equal to 0.2.³² The large cumulative relative deviations under enhanced naïve expectations are confirmed in this table, which increases rapidly for a larger debt-to-output ratio and for a higher tax rate. The differences between simple and enhanced fundamentalist expectations are indeed comparable with each other between the different regimes. Note that the cumulative deviations of the capital stock are even smaller under enhanced fundamentalist expectations for a larger value of debt-to-output ratio and a larger value for the tax rate, compared to the simple fundamentalist expectations. Taking past growth rates into account in expectations formation stabilises the capital stock, but not the GDP.

5.3 Stability of expectations and adjustment to shocks

After having discussed the properties of all expectation rules in relation to the stability and adjustment to shocks, some discussion points are interesting. The stability analysis of section 4 and the analysis on the speed of adjustment to shocks carried out in section 5 point at different features of the role of expectation rules in SFC models.

In the previous section we analysed the space of fiscal policy under different rules. We found that when people hold perfect foresight beliefs and stationary expectations, the stability of the economy is the smallest. However, as people move toward fundamentalist and naïve expectations, the stability of the economy is achieved under a wider room for fiscal policy.

In this section we complement this picture with the dynamics of adjustment after shocks. Naïve expectations, although more stable in the fiscal space, induce long persistence, making these expectations responsible for the longest adjustment to shocks among the rules we studied. Consistently with this intuition, stationary beliefs, although less stable in this space, cause a faster adjustment to economic shocks. This is consistent with the finding in economic literature that forward looking rules often lead to a faster adjustment to shocks than backward looking, naïve rules (Chiarella et al., 2006; Gasteiger, 2021).

³² For larger values of Φ , the economy becomes unstable if the debt-to-output ratio is increase from 40% to 50%.

Interestingly, the ‘educated dynamism’ of fundamentalist expectations contribute to economies which both are relatively stable (since people indeed converge towards a stationary forecast in due time), and provide a relatively quick adjustment period. This makes the fundamentalist rule quite interesting from a policy perspective.

These findings do also hold for the enhanced forms of fundamentalist and naïve expectations.

Finally, for adaptive expectations, we find both with respect to its impact on the stability of the economy, and with respect to the adjustment process intermediate results between naïve and fundamental expectations. Then region of stability is consistently smaller than that of fundamentalist expectations, and for small and moderate values of the tax rate it is also smaller than the region under naïve expectations. On the other hand, the adjustment to shocks under adaptive expectations is consistently more persistent when compared to fundamentalist expectations, but consistently less persistent when compared to naïve expectations. This observation emphasises the interesting nature of fundamentalist expectations.

6 Conclusion

Our analysis above illustrates that in a simple SFC model, the economy under different forms of expectations always converges to the stationary state levels of output and capital if it is stable. However, the stability of the model is highly dependent on the nature of the expectations. While after a shock the economy may be stable under one type of expectations, it can be highly unstable under another type of expectations. This implies that expectations should not be introduced in SFC-models in an ad hoc way, without much motivation, as usually is the case. It is important to understand the nature of expectations and to model these in a proper way.

In this analysis we show how the region of stability for government policy, defined by the combination of tax rate and debt-to-output ratio, is larger, the higher the path dependency in the economy is. We find that for low and intermediate tax rates naïve expectations, only depending on output in the previous period, lead to the largest region of stability. However, fundamentalist expectations also turn out to induce a quite large region of stability, exceeding that of adaptive expectations. Stationary expectations are associated with the smallest region of stability. A surprising finding is that “perfect foresight”, which usually is associated with a very stable economy, turns out to be highly unstable.

With respect to the dynamics of the economy in reaction to shocks under different types of expectations we find that naïve expectations induce strong persistence, which implies that the adjustment process is rather slow. Both fundamentalist and stationary expectations lead to a relatively fast adjustment process, while the adjustment process under adaptive expectations is somewhat slower.

We conclude that considering both the region of stability and the adjustment process after shocks an economy characterised by fundamental expectations is very interesting from a policy perspective. An interesting policy question then would be whether expectations could be nudged in the fundamentalist direction.

However, we should be careful to draw this kind of policy conclusions from such a simple model. A caveat of our analysis, is for instance that it is restricted to the moderate open economy. That is, foreign reserves can grow unlimited, without affecting consumption and investment. In an extended version of the model, we should include the impact of foreign reserves in a more sophisticated way.

A final point is that our analysis illustrates that when fiscal policy is defined by the combination of tax rate and debt-to-output ratio, as is the case in many Euro countries, severe shocks in the economy will always lead to instability. To deal with these shocks, this type of fiscal policy is not suitable and 'unconventional measures' have to be taken. In that sense the region of stability in our analysis reminds us of Leyonhufvud's corridor of stability (Leijonhufvud, 1973, 2009).

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Appendix A The model

A.1 The model of the full open economy

'Behavioural equations' are ignored. Instead of equations (7.1) – (7.4) we use:

$C_s = C_d = C$; $I_s = I_d = I$; $N_s = N_d = N$; $\Delta L_s = \Delta L_d = \Delta L$ (and ignore further distinctions s and d).

Moreover, we use the balance sheet identities: $L = M - R = K$

We follow G&L from equation (7.5):³³

Transactions firms

$$Y = C + I + G + X - M \quad (7.5') \quad \text{Gov. exp. } G \text{ and net exports } X - M \text{ added}$$

$$WB = Y - r_{l-1}L_{-1} - DA \quad (7.6)$$

$$\Delta L = I - DA \quad (7.8)$$

Transactions households

$$YD = WB + r_{m-1}M_{-1} + r_{r-1}B_{-1} + r_{R-1}R_{-1} - T \quad (7.9) \quad \text{Interest on } B \text{ \& } R, \text{ and taxes added}$$

$$\Delta M + \Delta B + \Delta R = YD - C \quad (7.10') \quad \text{Bond and reserves added}$$

Transactions banks

$$\Delta M = \Delta L \quad (7.11)$$

$$r_m = r_l \quad (7.12)$$

Transactions government

$$G = \Delta B + T - r_{B-1}B_{-1} \quad (7.13)$$

The wage bill

$$N = Y/pr \quad (7.14)$$

$$W = WB/N \quad (7.15)$$

Household behaviour

$$C = \alpha_1 YD^e + \alpha_2 (M_{-1} + B_{-1} + R_{-1}) + C^A \quad (7.16') \quad \text{Bonds and reserves added}$$

Firm behaviour

$$\Delta K = I - DA \quad (7.17)$$

$$DA = \delta \cdot K_{-1} \quad (7.18)$$

$$KT = \kappa \cdot Y^e \quad (7.19')$$

$$I = \gamma \cdot (KT - K_{-1}) + DA \quad (7.20)$$

New equations:

Government behaviour

$$T = \tau \cdot Y \quad (7.G2)$$

$$B = \beta \cdot Y \quad (7.G3)$$

$$r = r_l \quad (7.G4)$$

Bank behaviour

$$r_l = r^* \quad (7.21)$$

Foreign behaviour

$$X = X^A \quad (7.F1)$$

$$IM = \mu \cdot Y \quad (7.F2)$$

$$\Delta R = X - Im + r_{R-1}R_{-1} \quad (7.F3)$$

We find the model of Ch. 7 when $X^A = \beta = \tau = \mu = 0$

³³ The equation numbers are consistent with those of G&L, modifications are indicated.

A.2 The (implicit) role of other financial institutions

As we mentioned in section 2.1, the assumption that foreign reserves are held directly by households is a short-cut to keep the story of the model simple. Actually we assume that households lend F to other financial institutions, such as pension funds and funds holding non-traded assets, at a rate r . These financial institutions then hold foreign reserves R , also at a rate r . Since the reserves yield a return $r_{-1}R_{-1}$ and households receive $r_{-1}F_{-1}$, savings of financial institutions are zero and households essentially receive $r_{-1}R_{-1}$. Hence the short-cut, and Tables 1A and 2A collapse to Tables 1 and 2 in the text.

Table 1A The balance sheet

	Households	Firms	Banks	Fin.Instit.	Government	Abroad	Total
Deposits	+M		-M				0
Bonds	+B				-B		0
Loans		-L	L				0
Fixed Capital		+K					+K
Fin. Ass.	+F			-F			
Reserves				+R		-R	0
Wealth	-Vh	0	0	0 (-Vf)*	-Vg	-Va	-Vh-Vg-Va (-Vf)*
Total	0	0	0	0	0	0	0

Table 2A The accumulation of savings and investment matrix

	Households	Firms	Banks	Fin.Instit.	Gov.	Abroad	Total
Consumption	-C	+C+G			-G		0
Investment		+I					+I
Net Exports		+(X-M)				-(X-M)	0
Wages	WB	-WB					0
Taxes	-T				+T		0
Cons of fixed cap.		$-\delta \cdot K_{-1}$					$-\delta \cdot K_{-1}$
Interest	$+r_{-1}B_{-1}$ $+r_{-1}M_{-1}$ $+r_{-1}F_{-1} (0)^*$	$-r_{-1}L_{-1}$	$+r_{-1}L_{-1}$ $-r_{-1}M_{-1}$	$+r_{-1}R_{-1}$ $-r_{-1}F_{-1} (0)^*$	$-r_{-1}B_{-1}$	$-r_{-1}R_{-1}$	0
[Net savings]	[Sh]	[0]	[0]	[0 (Sfi)]	[Sg]	[Sa]	[Stot]
Δ Loans		$+\Delta L$	$-\Delta L$				0
Δ Deposits	$-\Delta M$		$+\Delta M$				0
Δ Bonds	$-\Delta B$				$+\Delta B$		0
Δ Oth.Fin.Ass	$-\Delta F (0)^*$			$\Delta F (0)^*$			
Δ Reserves				$-\Delta R$		$+\Delta R$	0
Δ Capital		$-\Delta K$					$-\Delta K$
Total	0	0	0	0	0	0	0

* In the moderate economy, the term between brackets does hold

The situation is different in the moderate economy. Here households do no longer receive interest income on their lending to financial institutions and F is not perceived as wealth in their consumption behaviour. As we explained above, this is quite similar to the German and the Dutch situation.

The consequence is that reserves can accumulate, without affecting output and capital, as we explained above. The reason is that the reserves remain within the financial institutions and increase their wealth position.³⁴

A.3 The model of the moderate open economy

We use the balance sheet identities: $L = M - F = K; F = R$

Transactions firms

$$Y = C + I + G + X - M \quad (7.5')$$

$$WB = Y - r_{l-1}L_{-1} - DA \quad (7.6)$$

$$\Delta L = I - DA \quad (7.8)$$

Transactions households

$$YD = WB + r_{m-1}M_{-1} + r_{-1}B_{-1} - T \quad (7.9)$$

No interest from R

$$\Delta M + \Delta B = YD - C \quad (7.10')$$

No accumulation of R (or F)

Transactions banks

$$\Delta M = \Delta L \quad (7.11)$$

$$r_m = r_l \quad (7.12)$$

Transactions government

$$G = \Delta B + T - r_{-1}B_{-1} \quad (7.13)$$

The wage bill

$$N = Y/pr \quad (7.14)$$

$$W = WB/N \quad (7.15)$$

Household behaviour

$$C = \alpha_1 YD + \alpha_2 (M_{-1} + B_{-1}) + C^A \quad (7.16')$$

No impact from R (or F)

Firm behaviour

$$\Delta K = I - DA \quad (7.17)$$

$$DA = \delta \cdot K_{-1} \quad (7.18)$$

$$KT = \kappa \cdot Y_{-1} \quad (7.19')$$

$$I = \gamma \cdot (KT - K_{-1}) + DA \quad (7.20)$$

New equations:

Government behaviour

$$T = \tau \cdot Y \quad (7.G2)$$

$$B = \beta \cdot Y \quad (7.G3)$$

$$r = r_l \quad (7.G4)$$

Bank behaviour

$$r_l = r^* \quad (7.21)$$

Other financial institutions

$$S_{fi} = r_{-1}R_{-1} \quad (7.B1)$$

Savings of other fin. Instit. added

Foreign behaviour

$$X = X^A \quad (7.F1)$$

$$IM = \mu \cdot Y \quad (7.F2)$$

$$\Delta R = X - Im + r_{-1}R_{-1} \quad (7.F3)$$

³⁴ A similar phenomenon happened with Dutch pension funds, see (Muysken & Meijers, 2023).

Appendix B The solution of the model for the moderate economy

B.1 The solution under simple expectations

The system of equations (48) – (49) has two eigenvalues and stability requires that the modulus of the eigenvalues are less than unity. Eigenvalues are determined by setting the determinant of the characteristic polynomial equal to zero:

$$\left| \text{Det} \begin{pmatrix} a - \lambda & b \\ c & d - \lambda \end{pmatrix} \right| = 0 \quad (\text{B1})$$

Solving for λ gives:

$$\lambda_{1,2} = \frac{a+d}{2} \pm \sqrt{\left(\frac{a+d}{2}\right)^2 - (ad - cb)} \quad (\text{B2})$$

If the term in the square root is positive, we can solve both eigenvalues using this equation. If the term is negative, one can find the modulus of the complex number.

In the latter case, i.e. $\left(\frac{a+d}{2}\right)^2 - (ad - cb) < 0$, we can write the root as a complex number as:

$$\lambda_{1,2} = x + yi = \frac{a+d}{2} \pm \left(\sqrt{ad - cb - \left(\frac{a+d}{2}\right)^2} \right) \cdot i \quad (\text{B3})$$

which implies that the eigenvalue becomes:³⁵

$$\lambda = \sqrt{\left(\frac{a+d}{2}\right)^2 + ad - cb - \left(\frac{a+d}{2}\right)^2} = \sqrt{ad - cb} \quad (\text{B4})$$

As a consequence stability requires that:

$$\text{Max} \left[\sqrt{ad - cb} ; \left| \frac{a+d}{2} \pm \sqrt{\left(\frac{a+d}{2}\right)^2 - (ad - cb)} \right| \right] < 1 \quad (\text{B5})$$

B.2 The solution under enhanced expectations

Due to enhanced expectations, see equations (16N') and (16F'), we add a second lag term as in the system of equations (48) – (49), which implies:

$$\begin{pmatrix} y \\ k \end{pmatrix} = \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} \begin{pmatrix} y_{-1} \\ y_{-2} \\ k_{-1} \end{pmatrix} \quad (\text{B6})$$

Let $x1_t = y_t$ and $x2_t = y_{t-1}$ so that $x1_{t-1} = y_{t-1} = x2_t$. Then (B.6) can be rewritten as:

$$\begin{pmatrix} x1 \\ x2 \\ k \end{pmatrix} = \begin{pmatrix} a & b & c \\ 1 & 0 & 0 \\ d & e & f \end{pmatrix} \begin{pmatrix} x1_{-1} \\ x2_{-1} \\ k_{-1} \end{pmatrix} \quad (\text{B7})$$

Stability conditions for this “simple” system gives three eigenvalues³⁶:

³⁵ The modulus of a complex number $x + yi$ is equal to: $\sqrt{x^2 + y^2}$.

³⁶ I accidentally switched rows 2 and 3, which of course does not give different eigenvalues, but different notations.

$$EV1 = \frac{a}{3} + \frac{e}{3} + \frac{A+Z^{\frac{2}{3}}}{\frac{1}{Z^{\frac{1}{3}}}} \quad (B8a)$$

$$EV2 = \frac{a}{3} + \frac{e}{3} - \frac{A-Z^{\frac{2}{3}}}{2*Z^{\frac{1}{3}}} - 1^{\frac{3}{4}} * \left(\frac{A-Z^{\frac{2}{3}}}{\frac{1}{Z^{\frac{1}{3}}}} \right) * 1i \quad (B8b)$$

$$EV3 = \frac{a}{3} + \frac{e}{3} - \frac{A-Z^{\frac{2}{3}}}{2*Z^{\frac{1}{3}}} + 1^{\frac{3}{4}} * \left(\frac{A-Z^{\frac{2}{3}}}{\frac{1}{Z^{\frac{1}{3}}}} \right) * 1i \quad (B8c)$$

with:

$$A = \frac{c}{3} - \frac{a*e}{3} + \frac{b*d}{3} + \frac{(a+e)^2}{9}; Z = \left(C + ((C+B)^2 - (A)^3)^{\frac{1}{2}} + B \right); C = \frac{b*f}{2} - \frac{c*e}{2}$$

$$B = \frac{(a+e)^3}{27} + \frac{(a+e)*(c-a*e+b*d)}{6} = \left(\frac{a+e}{3} \right) * \left[\left(\frac{a+e}{3} \right)^2 + \frac{(c-a*e+b*d)}{2} \right]$$

Note that EV2 and EV3 have a structure like $u \pm vi$ and that the last term is a complex term, so the eigenvalues of these are $\sqrt{u^2 \pm v^2}$.