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FMM WORKING PAPER

No. 33 ·September, 2018 · Hans-Böckler-Stiftung

THE ECONOMICS OF THE SUPER-MULTIPLIER: A COMPREHENSIVE TREATMENT WITH LABOR MARKETS

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ABSTRACT

This paper links the super-multiplier to Keynesian macroeconomics, showing it to be the most Keynesian of growth perspectives. Next, the paper shows that the super-multiplier is a micro-economically coherent theory of investment and capital accumulation. Firms' decisions regarding capital accumulation coordinate demand and supply growth in goods markets. The paper then explores the implications of incorporating the super-multiplier in the Neo-Kaleckian and Cambridge growth models. Lastly, it shows how labor markets and unemployment can be added into super-multiplier models to provide a comprehensive growth model that addresses Solow's (1956) labor market knife-edge problem. Incorporating labor markets does not change the fundamental super-multiplier result that growth is determined by the growth of autonomous demand.

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Keywords: Super-multiplier, growth, unemployment, endogenous technical progress, Solow Knife-edge, Hicks, Kaldor.

JEL ref.: O4, O41, O33, E12.

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August, 2018

1. Introduction

Recently, there has been a surge of interest among Post Keynesian economists in the super-multiplier as a basis for framing Keynesian growth models. The concept of the super-multiplier dates back to Hicks' (1950) work on the trade cycle in the 1950s. As shown by Cesaratto (2017), it was applied to a growth context by Ackley (1961, 1963) in the early 1960s in his macroeconomics textbook and his econometric study of the Italian economy. However, it then lay essentially dormant until it was revived by Sraffian economists in the 1990s and 2000s (Serrano, 1995, Bortis, 1997, Cesaratto et al., 2003). More recently, it has been significantly invigorated by contributions from Freitas and

Serrano (2015), Allain (2015), Lavoie (2016), Serrano and Freitas (2017), and Dejuan (2017).

The Sraffian revival often talks of the super-multiplier as a Sraffian closure to growth models. However, while it is consistent with the Sraffian approach to production and distribution, this paper shows the super-multiplier is a theory of the rate of capital accumulation which can in principle be applied in multiple different Post Keynesian growth frameworks.

The existing literature on the super-multiplier is complex and mathematical. The paper aims to make it more transparent by clarifying the nature and possible applications of the super-multiplier. It begins by linking the super-multiplier to Keynesian macroeconomics and, in many ways, the super-multiplier is the most Keynesian of Keynesian growth models. The intuition behind the super-multiplier follows directly from the income-expenditure model, a model which captures the most fundamental of Keynesian insights.

Next, the paper shows that the super-multiplier is a micro-economically coherent theory of investment and capital accumulation. Firms' decisions regarding capital accumulation coordinate demand and supply growth in goods markets. Thereafter, the paper shows how the super-multiplier can be incorporated in both neo-Kaleckian and Cambridge growth models, and it explores the implications of doing so. The paper's object is not to argue for one specification over another, but rather to show the generality of the super-multiplier approach to capital accumulation and growth.

Lastly, the paper shows how labor markets and unemployment can be added into super-multiplier models to provide a comprehensive growth model that addresses Solow's

(1956) unemployment knife-edge problem in a Harrodian world with fixed proportions production. Incorporating labor markets does not change the fundamental super-multiplier result that growth is determined exclusively by the growth of autonomous demand.

2. Keynesian macroeconomics and the logic of the super-multiplier

The logic of the super-multiplier model follows directly from the principles of Keynesian macroeconomics and can be illustrated by the simple income – expenditure model. The equations of that model are given by

(1) $Y = D$

(2) $D = A + bY$ $\qquad\qquad A > 0,\ 0 < b < 1$

Y = real output, D = real aggregate demand, A = autonomous spending, b = marginal propensity to spend. Equation (1) is the goods market clearing condition. Equation (2) is the aggregate demand (AD) function.

Solving equations (1) and (2) yields

(3) $Y = D = A/[1 - b]$

The logic of the model is that output adjusts to equal AD. AD is in turn equal to autonomous expenditure multiplied by the expenditure multiplier ($1/[1 - b]$).

Using equation (3), the growth rate relationship is given by

(4) $g_Y = g_D = g_A$

g_Y = growth of output, g_D = growth of AD, g_A = growth of autonomous demand. A simple extrapolation of the fundamental Keynesian model to a growth context yields the claim that the growth of output is determined by the growth of AD, which in turn equals the growth of autonomous demand. If Keynesian macroeconomic logic rules in the long run, the growth of autonomous demand rules the roost and determines the growth rate.

This simple income-expenditure framing spotlights a critical Keynesian distinction between the level of economic activity (Y) and the growth rate (g_Y).The level of macroeconomic activity is the terrain of macroeconomics. The rate of growth of output is the terrain of growth theory. There should be a consistent theoretical bridge between the two. As will be shown, the super-multiplier provides one such bridge.

3. Microeconomics: the super-multiplier as a theory of investment

Extrapolation of the income – expenditure model to a growth context provides the basic intuition as to why growth is ruled by the growth of autonomous demand. However, over time the economy must accumulate the productive capital needed to produce the output needed to meet demand. The supply side must be consistent with the demand side for a coherent theory of growth.

The connection from the demand side to capital accumulation runs via the production process, which is described by the following production function:

(4) $Y = \text{Min}[K/v, L/b]$

K = capital, v = capital-output ratio , L = effective labor input, b = labor-output ratio.

Assuming the economy is unconstrained by labor supply, output is given by

(5) $Y = K/v$ $\qquad v > 1$

Expressing equation (5) as a growth relation yields the supply side condition:

(6) $g_Y = g_K$

g_K = growth of the capital stock. Output can only grow at the rate of capital accumulation.

Assuming that firms are at their desired level of output, each period they need to add sufficient capital such that capacity expands to meet the expected increase in demand. From the production function, this implies:

$$(7)\ I = dK = vE[dD]$$

I = investment, dK = change in the capital stock, $E[dD]$ = expected increase in demand. The additional capital that must be added is the capital-output ratio multiplied by the expected increase in demand.

If firms are not at desired output equation (7) must be augmented by an additional disequilibrium term reflecting that gap, and it is given by:

$$(7.a)\ I = dK = vE[dD] + \gamma(v[Y^* - Y]) \qquad \gamma' > 0,\ \gamma(0) = 0$$

Y^* = desired output. The additional term reflects that fact that not only must firms adjust the capital stock to meet the expected change in demand, they must also adjust the capital stock to bring it in line with desired output at the existing level of demand.[1] The term $v[Y^* - Y]$ determines the capital stock disequilibrium at the current level of output.

The relation between capital and output explains the power of changes in expected demand to stimulate macroeconomic activity, via investment spending. To meet one unit of additional demand, firms must add v units of capital, with v being the capital - output ratio. Since $v > 1$, firms must add more capital than the amount of additional demand they aim to satisfy. One way of thinking about this is via a Bőhm-Bawerk roundabout production perspective, the amount of output embodied in a unit of investment is a large multiple of the output that unit produces each period. Consequently, increased expected demand gives a strong boost to current demand via capital accumulation, which is another reason (in addition to investment spending volatility) why investment spending is so important in Keynesian macroeconomics.

Dividing equation (7) by K and using the goods market clearing condition (Y = D)

[1] Equation (7.a) has a non-linear disequilibrium adjustment mechanism, allowing for the possibility of more robust adjustment the further firms are from their desired output level.

yields the rate of capital accumulation, which is given by

(8) $I/K = g_K = g^{ED}$

g^{ED} = firms' expected rate of demand growth. Investment spending is guided by the expected increase in demand, thereby linking the demand side of the economy to the supply side. The microeconomic logic is simple. The production function requires firms must grow their capital stock at the rate of expected demand growth to be able to supply future demand. Reconciliation of the demand side ($g_Y = g^{ED}$) with the supply side ($g_Y = g_K$) is therefore achieved via investment spending, which coordinates supply growth with demand growth.

For the case where firms respond to deviations from normal capacity utilization, the rate of capital accumulation is given by

(9) $I/K = g_K = g^{ED} + \gamma(u^* - u) \qquad \gamma' > 0, \gamma(0) = 0$

u^* = normal capacity utilization, u = actual capacity utilization.

The inclusion of a capacity utilization disequilibrium term then raises issues of disequilibrium dynamics and Harrodian instability. Harrodian instability concerns the possibility that the evolution of firms' expectations of demand growth may cause instability. Specifically, as firms try to invest and close the gap between desired and actual capacity utilization, that will stimulate economic activity and may cause an unstable revision of expected demand growth.

That raises the question of how expectations of demand growth are formed. If firms believe in the Keynesian macro model and use it to guide their thinking about the economy, then the rational expectation of expected demand growth is given by

(10) $E[g^{ED}] = g_A$

The combination of the Keynesian macro model and rational expectations (RE) therefore solves the problem of Harrodian instability which could arise from destabilizing revisions of expected demand growth. The Keynesian RE framing also provides a microeconomic justification for the assumption that expected demand growth equals autonomous demand growth (Bortis, 1997; DeJuan, 2017).

The above microeconomic derivation of the accumulation process is revealing of three issues. First, "capacity creating" demand growth (g_K) and "non-capacity creating" demand growth (g_A) are related, with the latter determining the equilibrium rate of capacity creating demand growth. Capital is not accumulated for its own sake. Instead, the demand for capital is a derived demand, reflecting the fact that new capital is installed to satisfy future demand. The growth effect of non-capacity creating demand shows up through its impact on capital accumulation, and not as a separate effect.

Second, although the Harrodian instability problem is resolved by a Keynesian RE frame, there remains the issue of stability of the macroeconomic accumulation process. As shown below, that requires the Keynesian stability condition also be satisfied.

Third, recent literature on the super-multiplier (see for example Allain (2015), Freitas and Serrano (2015), Lavoie (2016)) is easily misinterpreted. It is not non-capacity creating demand growth *per se* that stabilizes the super-multiplier model. Rather, the model is stable because expectations of demand growth are stable so that the condition for Harrodian stability is met, and the Keynesian stability condition is also met.

4. The simplest super-multiplier model

The above logic of investment and capital accumulation can now be placed in a simple growth model to illustrate the logic of the super-multiplier. The model is given by

(11) $I/K = I = g_K = g^{ED}$

(12) $g^{ED} = g_A$

(13) $S/K = S = I/K$

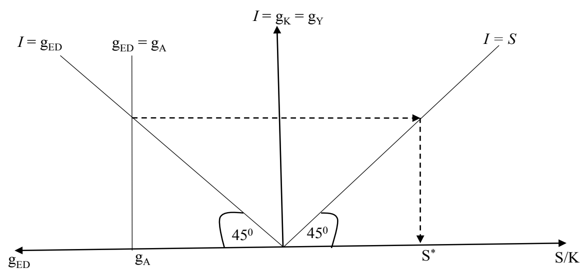
(14) $g_Y = g_K$

I = rate of capital accumulation, S = saving rate. Equation (11) is the investment function in which the accumulation rate equals expected demand growth.[2] Equation (12) has expected demand growth equal to autonomous demand growth. Equation (13) is the goods market clearing condition. Equation (14) has output growth equal to the rate of capital accumulation.

The model is illustrated in Figure 1. The left-hand panel has autonomous demand growth determining expected demand growth, which in turn determines the rate of capital accumulation and growth. The right-hand panel has the saving rate passively adjust to the rate of capital accumulation. The implication is autonomous demand growth rules the roost and determines the growth rate.

[2] Recalling the discussion in the previous section, this assumes firms are at their desired level of capacity utilization.

Figure 1. The simple super-multiplier model



There are three pieces to the model. First, expected demand growth is determined by autonomous demand growth via the logic of the Keynesian income-expenditure model. Second, investment serves to coordinate supply and demand growth, with firms adding the capacity needed to meet expected demand growth. Third, the goods market clears via the saving – investment equilibrium condition. In this simplest of models, there is no independent saving function and goods market balance is achieved by saving passively adjusting to investment.

5. A neo-Kaleckian super-multiplier model

The above simple model assumed saving passively accommodated investment. That assumption side-steps the fundamental concern of Keynesian economics regarding the coordination of saving and investment, and the question of how they are brought into balance. This section presents a super-multiplier model in which saving is not passive and is impacted by the distribution of income in accordance with Post Keynesian theory. The model is termed neo-Kaleckian because capacity utilization is variable. The model also makes clear that the super-multiplier is a theory of capital accumulation, which can be applied in different theoretical frameworks.

The new model involves adding the following three equations to the system given by equations (11) – (14):

(15) $S = -a + s_W\sigma_W u + s_\Pi\sigma_\Pi u$ $\qquad 0 < s_W < s_\Pi < 1$

(16) $a = A/K$

(17) $\sigma_W + \sigma_\Pi = 1$ $\qquad 0 < \sigma_W < 1, \quad 0 < \sigma_\Pi < 1$

a = autonomous spending per unit of capital, s_W = propensity to save out of wage income, s_Π = propensity to save out of profit income, σ_W = wage share of income, s_Π = profit share of income. Equation (14) determines the saving rate as a function of income distribution, with a higher profit share raising saving. Autonomous spending constitutes dis-saving. Equation (15) defines the autonomous spending rate. Equation (17) is the income share adding up constraint. The endogenous variables are g_K, g^{ED}, g_Y, I, S, and u.

The model now has an independent saving function, and goods market balance (i.e. saving – investment balance) is achieved by adjustment of capacity utilization. However, the rate of accumulation depends exclusively on expected demand growth, implying firms are at their desired capacity utilization. In the neo-Kaleckian super-multiplier model, capacity utilization is variable and firms are always at desired capacity utilization, so it is as if every rate of capacity utilization is optimal.

The solutions to the model are given by

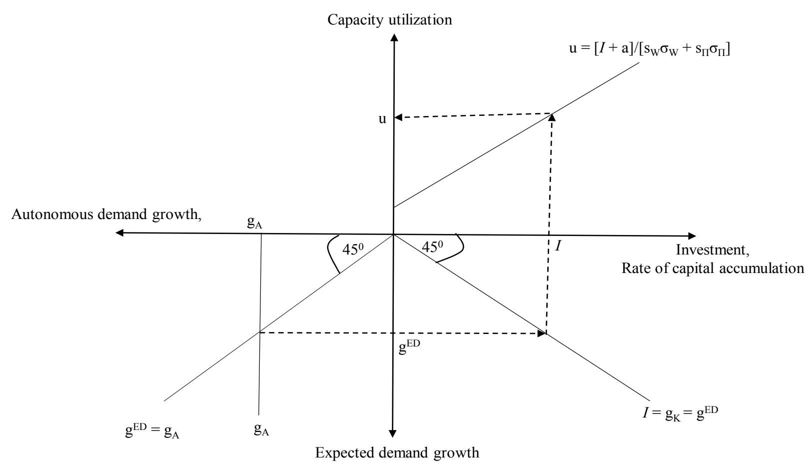
(18) $g_Y = g_K = g_A$

(19) $u = [g_A + a]/[s_W\sigma_W + s_\Pi\sigma_\Pi]$

Increases in the propensities to save lower utilization (du/ds$_W$ < 0, du/ds$_\Pi$ < 0), as does an increase in the profit share (du/$d\sigma_\Pi$ < 0). An increase in the wage share raises utilization (du/$d\sigma_W$ > 0).

The model is illustrated in Figure 2. The southwest panel has autonomous demand growth determining expected demand growth. The southeast panel has expected demand growth determining the rate of capital accumulation (i.e. the investment rate). The northeast panel is the goods market equilibrium (IS) schedule, and has capacity utilization adjusting to equalize saving and investment.

Figure 2. The neo-Kaleckian super-multiplier model.



As in the simple model, autonomous demand growth rules the roost and determines the rate of capital accumulation and growth. Saving has no impact on accumulation and growth, but it is not passive or neutral. A higher saving rate shifts the IS schedule down, lowering capacity utilization. The reverse holds for a lower saving rate. The logic is that higher saving means a lower utilization rate is warranted to maintain goods market equilibrium.

A second feature of the model, related to Figure 1, is the distinction between the steady state share of autonomous spending (a) and the rate of growth of autonomous spending (g_A). For growth, the only thing that matters is g_A. However, increases in "a" and g_A both increase capacity utilization. Higher "a" raises the autonomous component of AD. Higher g_A spurs investment which raises AD.

6. A quasi neo-Kaleckian super-multiplier with fixed normal capacity utilization

The previous model had steady-state capacity utilization being variable. This section presents a quasi neo-Kaleckian super-multiplier model. The model is neo-Kaleckian in the sense that capacity utilization is the variable that adjusts in response to dis-equilibrating disturbances and adjusts at each moment in time to ensure goods market balance. It is quasi neo-Kaleckian because the economy has a long run fixed normal capacity utilization rate.[3]

If capacity utilization gravitates back to normal, then there is need for an adjustment process that restores the normal rate. Serrano and Freitas (2017, p.75) have utilization reverting to normal via a smooth disequilibrium gradual adjustment mechanism. However, that mechanism applies to "state" variables, which is inconsistent with the underlying model in which capacity utilization is a "jump" variable that instantaneously clears the goods market at each moment in time.

Lavoie (2016) presents an alternative mechanism that focuses on adjustments in the share of autonomous expenditures (a = A/K), the growth of which is the driver of capital accumulation. The evolution of the share of autonomous demand is given by

(20) $g_a = g_A - g_Y = g_A - g_K$

The steady state condition is $g_a = 0$, which ensures a constant share of autonomous demand.

[3] Schoder (2014) reports empirical evidence from the US economy supportive of this pattern.

The investment function can then be augmented to incorporate linearized disequilibrium capacity utilization effects as follows:

(21) $I = g_K = g^{ED} + \gamma[u - u^*] \qquad \gamma > 0$

u^* = normal capacity utilization. The accumulation rate therefore responds positively to above normal utilization, and negatively to below normal utilization. This specification allows firms to be away from their desired rate of capacity utilization.

Utilization is now a jump variable that adjusts to clear the goods market. Equating saving and investment (equations (15) and (21)) and solving yields the instantaneous rate of capacity utilization, which is given by

(22) $u = [g^{ED} + a - \gamma u^*]/[s_W\sigma_W + s_\Pi\sigma_\Pi - \gamma]$

Substituting the expression for capacity utilization into equation (21) and substituting the resulting equation into equation (20) yields:

(23) $g_a = g_A - g^{ED} - \gamma[g^{ED} + a - \gamma u^*]/[s_W\sigma_W + s_\Pi\sigma_\Pi - \gamma] + \gamma u^*$

Since $g_A = g^{ED}$, equation (22) reduces to

(24) $g_a = \gamma u^* - \gamma[g_A + a - \gamma u^*]/[s_W\sigma_W + s_\Pi\sigma_\Pi - \gamma]$

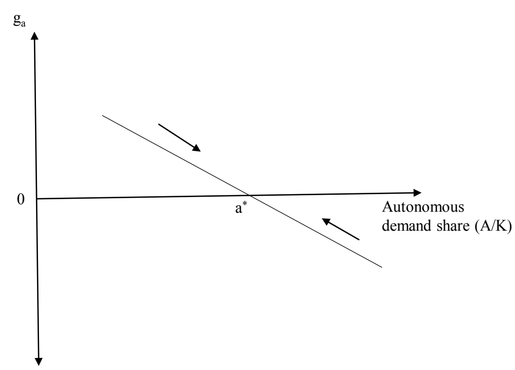
This is a first order differential equation governing the evolution of the autonomous demand share. The stability condition is $dg_a/da = -\gamma/[s_W\sigma_W + s_\Pi\sigma_\Pi - \gamma] < 0$. The numerator is positive, but the denominator can be negative if investment is more sensitive than saving to capacity utilization. An unstable outcome corresponds to a situation in which the Keynesian stability condition is violated because firms adjust investment spending too strongly in response to capacity disequilibrium, thereby exacerbating the disequilibrium.

Assuming Keynesian stability holds, the steady state autonomous demand share can be obtained by setting $g_a = 0$. Solving equation (23) then yields

(25) $a^* = u^* + [\gamma u^* - g_A]/[s_W\sigma_W + s_\Pi\sigma_\Pi - \gamma]$

Figure 3 shows the adjustment path.[4] In the short run, increased demand increases capacity utilization, but the economy gravitates back to normal utilization which is determined exogenously.[5] The steady state growth rate is unaffected by such increases, and remains determined exclusively by the rate of autonomous demand growth.

Figure 3. Stability of the neo-Kaleckian super-multiplier model with normal capacity utilization.



Equation (25) determining the steady state autonomous demand share is especially interesting. It shows that the steady state share is structurally constrained. If the economy is stable, a given growth of autonomous demand (g_A) will ultimately generate a particular autonomous demand share (a^*). Conversely, a given desired autonomous demand share requires a particular growth rate to sustain that share. A surprising feature of equation (24)

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[4] There are several requirements for the model to be stable and make economic sense. The Keynesian stability condition requires $s_W\sigma_W + s_\Pi\sigma_\Pi - \gamma v > 0$. The requirement that capacity utilization be positive then implies that $g_A + a - \gamma v u^* > 0$. Lastly, the requirement that $a^* > 0$ implies $u^* + [\gamma v u^* - g_A]/[s_W\sigma_W + s_\Pi\sigma_\Pi - \gamma v] > 0$.

[5] The proof that the economy gravitates back to u^* in equilibrium is as follows. In steady state $g_a = g_A - g_K = 0$. It is also the case that $g_A = g^{ED}$, which implies $g_K = g^{ED}$. From the investment function that implies $u = u^*$.

is the higher the growth rate of autonomous demand, the lower the steady state share. That is because a faster growth of autonomous demand increases the capital stock and output, causing the autonomous demand share to fall.

7. A Cambridge super-multiplier model

Just as the super-multiplier model can be fitted into a neo-Kaleckian frame, so too it can be fitted into a Cambridge frame. The key difference is the profit share does the adjusting instead of capacity utilization, and the latter is permanently fixed at the normal rate. This corresponds to the framework adopted by Bortis (1997).

The equations of the Cambridge super-multiplier model are identical to those of the neo-Kaleckian super-multiplier model, only the wage share (σ_W) and profit share (σ_Π) do the adjusting to ensure goods market balance instead of capacity utilization (u). The reduced form saving function becomes:

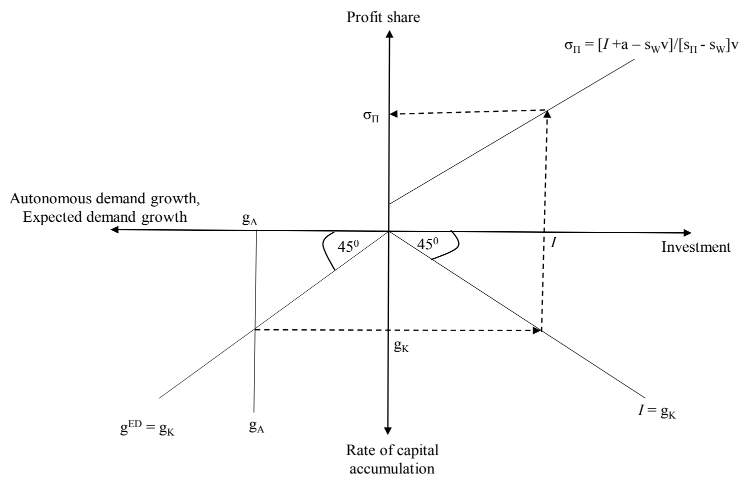
(26) $S = -a + s_W u^* + [s_\Pi - s_W]\sigma_\Pi u^*$

The solution for the profit share is

(27) $\sigma_\Pi = [I + a - s_W u^*]/[s_\Pi - s_W]u^*$

The model is illustrated in Figure 4. Increases in investment or the autonomous demand share increase AD. Given fixed normal capacity utilization, that increases the profit share in order to increase saving and equilibrate the goods market. Consequently, contrary to the claims of Sraffians, the super-multiplier model is fully consistent with endogenous income distribution. Equation (26) also implies there is a growth – distribution trade-off, with faster growth inducing a higher profit share.

Figure 4. The Cambridge super-multiplier model.



As with the neo-Kaleckian super-multiplier model, there is also room for disequilibrium analysis. However, instead of normal capacity utilization there is a normal profit rate, and disequilibrium manifests itself via departures from that normal profit rate. The linearized disequilibrium augmented investment function and profit rate are given by

(28) $I = g_K = g^{ED} + \gamma[\pi - \pi^*]$ $\qquad \gamma > 0$

(29) $\pi = \sigma_\Pi u^*$

π = profit rate, π^*= normal profit rate.

The instantaneous profit share is obtained by setting saving equal to investment and solving for the profit share, which yields

(30) $\sigma_\Pi = \{g^{ED} - \gamma\pi^* + a - s_W u^*\}/\{[s_\Pi - s_W] - \gamma\}u^*$

Faster demand growth (g^{ED}) and a higher autonomous spending share (a) both increase the profit share. The logic is they both increase demand which, given fixed capacity utilization, increases the profit share which then diminishes demand and restores goods market balance.

Appropriate substitution into the equation of motion for the autonomous demand share given by equation (20), yields:

(31) $g_a = g_A - g_K = -\gamma\pi^* + \gamma\{g^{ED} - \gamma\pi^* + a - s_W u^*\}/\{[s_\Pi - s_W] - \gamma\}$

The stability condition is $dg_a/da = -\gamma/\{[s_\Pi - s_W] - \gamma\} < 0$. The numerator is negative, but the denominator can be negative if investment is more sensitive than saving to the profit rate. Once again, there is the possibility of instability. The problem is the Keynesian stability condition may be violated because firms adjust investment spending too strongly in response to profit rate disequilibrium, thereby exacerbating the disequilibrium.

The steady state autonomous demand share (a) can be obtained by setting $g_a = 0$ and solving equation (31). The solution is given by

(32) $a^* = [s_\Pi - s_W]\pi^* + s_W u^* - g_A$

If the economy is stable, the logic of the adjustment mechanism is as follows. Increases in the share of autonomous appending or the growth rate of AD raise demand, which increases the profit rate. That sends a signal for faster capital accumulation, which in turn raises the economy's supply capacity and eventually brings the profit rate down to its normal rate. The gradual reversion of the profit rate to its normal rate then slowly reduces the rate of accumulation, which causes g_a to decelerate until the autonomous expenditure share is again stable.

8. Adding the labor market and unemployment

The microeconomics of the super-multiplier show how firms' capital accumulation decisions coordinate the demand side and supply side of goods markets. However, firms also need labor inputs, which calls for modelling the labor market. Like other Post

Keynesian growth models, the existing super-multiplier literature ignores the labor market and is afflicted by the Harrod (1939) - Solow (1956) labor market knife-edge problem. Unless labor supply growth exactly equals labor demand growth, the unemployment rate will either implode or explode. The problem is how is such balance achieved in a world with fixed proportions production, exogenously determined capital accumulation, and exogenously given labor force growth.

This section shows how a stable labor market with a stable steady state unemployment rate can be incorporated into the super-multiplier model. The mechanism for equalizing labor supply and demand growth is the Kaldor – Hicks technological progress function developed by Palley (2012, 2013, 2014). The attribution to Kaldor (1957) reflects the fact that technological progress is endogenous and impacted by the rate of capital accumulation. The attribution to Hicks (1932) reflects the fact that the state of the labor market impacts technological progress, with firms having a greater incentive to innovate when labor markets are tight and unemployment is low.[6]

Fazzari et al. (2018) have recently also used this mechanism. Their focus is the goods market and showing how aggregate supply accommodates aggregate demand growth, rather than showing how labor supply and demand growth balance. However, there is an implicit equivalence since behind aggregate supply lies the labor market and labor supply. The simplified framework below shows how all versions of the super-multiplier can be plugged into the Kaldor – Hicks mechanism, thereby connecting labor markets with the capital accumulation decisions of firms. That provides a comprehensive treatment of the super-multiplier from its goods market origins through to the labor market.

[6] In Palley (2012) labor-saving innovation is a positive function of the wage share, and the wage share is a negative function of the unemployment rate. The innovation effect of unemployment therefore works through income distribution.

The equations of the supply-side and the labor market are given by:

(33) $Y = \text{Min}[K/v, \mathit{L}/b]$

(34) $U = 1 - L/N$

(35) $g_Y = g_K = g_{\mathit{L}}$

(36) $g_K = I/K$

(37) $g_{\mathit{L}} = g_L + g_T$

(38) $g_U = g_N - g_L$

Y = output, K = capital, v = capital - output ratio, *L* = effective labor input, b = effective labor - output ratio, I = investment, S = saving, U = unemployment rate, L = employment, N = population, g_Y = output growth, g_K = growth of the capital stock, g_L = employment growth, g_N = labor force growth, g_T = rate of labor augmenting technical progress, g_U = rate of change of the unemployment rate.

Equation (33) is the aggregate production function which is Leontieff. Labor input equals effective labor, reflecting the assumption that technical progress is labor augmenting. Equation (34) is the definition of the unemployment rate. The unemployment rate depends on actual labor employed rather than effective labor employed. Equation (35) has output growth equal to the rate of capital accumulation, which in turn must equal the rate of growth of effective labor input growth given Leontieff technology. Equation (36) defines the growth of the capital stock which equals the rate of capital accumulation. Equation (37) defines the rate of effective labor input growth, which is equal to employment growth plus the rate of labor augmenting technical progress. Equation (38) determines is the rate of change of the unemployment rate, which is equal to labor force growth minus actual employment growth.

The Kaldor – Hicks technological progress function is the critical building block. It makes the rate of labor augmenting technical progress a negative function of the unemployment rate, and is given by:[7]

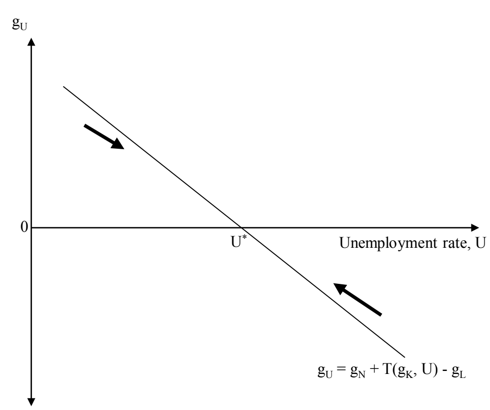
(39) $g_T = T(g_K, U)$ $\qquad T_{gK} > 0, T_U < 0$

Substituting equations (35), (37), and (39) into equation (38) yields

(40) $g_U = g_N + T(g_K, U) - g_K$

Equation (40) is a first-order differential equation and it governs the evolution of the unemployment rate. It is illustrated in Figure 5. The necessary condition for stability is $dg_U/dU < 0$. As the unemployment rate increases, the rate of increase decreases so that the unemployment rate eventually stabilizes. A key assumption is that $T_U < 0$.

Figure 5. The stable unemployment rate adjustment mechanism.



For simplicity, I assume a linear endogenous technical progress function given by

(41) $g_a = \alpha_0 + \alpha_1 g_K - \alpha_2 U$ $\qquad \alpha_0 > 0, \alpha_1 > 0\ \alpha_2 > 0$

[7] An equivalent specification is to make the rate of technical progress a positive function of the employment rate (E) since U = 1 – E. The only effect is to flip the sign of the partial derivative of the technical progress function with respect labor market conditions. In log linear models there is an advantage to using E as the argument.

Substituting equation (41) in equation (39) then yields a specific form fundamental differential equation given by

(42) $g_U = g_N + \alpha_0 - \alpha_2 U - [1 - \alpha_1]g_K$

The long run equilibrium solution is obtained by setting $g_U = 0$ and solving, which yields

(43) $U^* = \{g_N + \alpha_0 – [1 - \alpha_1]g_K\}/\alpha_2$

Incorporation of the super-multiplier is straightforward. Recall the rate of capital accumulation is given by $I = g_K = g^{ED}$. Substituting into equation (43) yields

(44) $U^* = \{g_N + \alpha_0 – [1 - \alpha_1]g^{ED}\}/\alpha_2$

Faster autonomous demand growth therefore yields a lower steady state unemployment along with faster output growth. The warranted rate (g^{ED}) also rules the roost since growth is unaffected by the natural rate (g_N). However, the natural rate does affect the steady state unemployment rate, and faster labor force growth increases the unemployment rate. That holds for all the models examined: the standard super-multiplier, the neo-Kaleckian super-multiplier without and normal capacity utilization, and the Cambridge super-multiplier without and with normal profit.

The above treatment assumes there are no disequilibrium capacity utilization or profit share effects on capital accumulation. That simplifies the presentation of the argument. If there are such disequilibrium effects, then the capital accumulation function must be appropriately modified as follows

(45) $U^* = \{g_N + \alpha_0 – [1 - \alpha_1]\{g^{ED} + \gamma[u – u^*]\}\}/\alpha_2$

If the Keynesian stability condition holds and the goods market is stable, capacity utilization reverts to normal and the labor market is also stable.

The model can be easily extended to allow the labor market to affect income distribution. For instance, a higher unemployment rate and weaker worker bargaining power may increase the profit share (i.e. lower the wage share) as follows:

(46) $s_\Pi = \sigma(U, \lambda)$ $\qquad$ $\sigma_U > 0, \sigma_\lambda < 0$

λ is a worker bargaining power variable. Given the income shares adding up constraint, that implies the wage share is a negative function of the unemployment rate and given by

(47) $s_W = 1 - \sigma(U, \lambda) = \Sigma(U, \lambda)$ $\qquad$ $\Sigma_U < 0, \Sigma_\lambda > 0$

Other specifications are also possible, including a concave profit share in which the profit share eventually turns lower as the unemployment rate falls.

This expanded labor market with income distribution effects can then be incorporated in the neo-Kaleckian super-multiplier model. However, it cannot be incorporated in the Cambridge super-multiplier because distribution in that model is determined outside the labor market. In the neo-Kaleckian model with variable capacity utilization, labor market distribution effects impact saving, which impacts capacity utilization. Lower unemployment or greater worker bargaining power increase the wage share, which increases capacity utilization. The reverse holds for higher unemployment and reduced worker bargaining power. However, there is no impact on steady-state growth which remains determined exclusively by autonomous demand growth in the super-multiplier framework.

In the neo-Kaleckian model with fixed normal capacity utilization, labor market distribution effects have no long run impact on utilization since the economy reverts to the normal utilization rate. However, there is an impact on equation (24) which determines the relationship between the autonomous demand share and the rate of growth of autonomous demand. That is because labor force growth impacts the unemployment rate, which impacts income distribution, which impacts the equilibrium autonomous demand share.

9. Conclusion: what determines the rate of capital accumulation?

This paper has shown that, in many ways, the super-multiplier is the most Keynesian of Keynesian growth models. Next, the paper showed that the super-multiplier is a micro-economically coherent theory of investment and capital accumulation. Firms' decisions regarding capital accumulation coordinate demand and supply growth in goods markets. Thereafter, it showed that the super-multiplier can be incorporated in multiple different growth frameworks. Lastly, it showed how labor markets can be combined with the super-multiplier to provide a comprehensive growth model that addresses the Harrod (1939) - Solow (1956) labor market knife-edge problem in economies with fixed proportions production. Incorporating labor markets does not change the fundamental super-multiplier result that growth is determined by autonomous demand growth.

It sometimes seems to be implied that the super-multiplier is an exclusively Sraffian concept, but it is consistent with multiple different approaches to growth. The super-multiplier model boils down to a debate about the theory of investment and capital accumulation. The super-multiplier makes the equilibrium rate of capital accumulation exclusively a function of expected demand growth. That contrasts with other theories which identify the profit rate, the profit share, and capacity utilization as important factors influencing accumulation.

The great unanswered question in super-multiplier theory is what determines the rate of growth of autonomous demand. If it is influenced by factors such as income distribution and the unemployment rate, then the super-multiplier model will effectively collapse back into a frame similar to other PK growth models – albeit via a path one derivative removed (i.e. the growth of autonomous demand function). That question provides material for future research.

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Impressum

Publisher: Hans-Böckler-Stiftung, Hans-Böckler-Straße 39, 40476 Düsseldorf, Germany
Contact: fmm@boeckler.de, www.fmm-macro.net

FMM Working Paper is an online publication series available at:
https://www.boeckler.de/imk_108537.htm

ISSN: 2512-8655

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