RECONSIDERING THE NATURAL RATE HYPOTHESIS

Robert Calvert Jump*, Engelbert Stockhammer**

ABSTRACT

The natural rate hypothesis states that there exists an unemployment rate at which inflation is stable, and that this unemployment rate is independent of aggregate demand shocks. The hysteresis hypothesis, in contrast, states that the long run unemployment rate can be affected by aggregate demand shocks. While policy makers have warned of the risk of hysteresis since the 2008 financial crash, hysteresis effects are not incorporated into the macroeconometric models used by policy making institutions. This paper presents Bayesian estimates of hysteresis effects using unobserved components models of the type used by the European Commission and OECD. We demonstrate that the posterior probability of the natural rate hypothesis holding in Germany, France, and the UK is very low, lending empirical support to the hysteresis hypothesis. We suggest that the models used by the European Commission and OECD should be amended to reflect policy makers' views on hysteresis.

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Reconsidering the natural rate hypothesis∗

Robert Calvert Jump†  Engbert Stockhammer‡

April 30, 2019

Abstract

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Keywords: Unemployment, Hysteresis, NAIRU, Business Cycles.

JEL Codes: E24, E60, E61.

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1 Introduction

The natural rate hypothesis states that there exists an unemployment rate at which inflation is stable, and that this unemployment rate is independent of aggregate demand shocks (Blanchard 2018). In particular, if monetary or fiscal policy targets an unemployment rate below the natural rate, inflation will increase until the policy is abandoned. The natural rate of unemployment is usually referred to as the NAIRU, or non-accelerating inflation rate of unemployment, which emphasises its role as a practical constraint on aggregate demand management (Stiglitz 1997; Ball and Mankiw 2002). Using this terminology, the natural rate hypothesis states that there exists a NAIRU, and that the NAIRU is independent of aggregate demand shocks.

The major competitor to the natural rate hypothesis is the hysteresis hypothesis. This was initially proposed to explain the persistent increases in European unemployment rates observed during the 1980s, and is firmly back on the policy agenda since the 2008 financial crisis. Thus the governor of the Bank of England, in a speech at the Bank of International Settlements (Carney 2014), observed that,

. . . to the extent appropriate under our mandates, the monetary policy response has represented a race against long-term (or hysteretic) unemployment. As Janet Yellen remarked, “the risk that continued high unemployment could eventually lead to more-persistent structural problems underscores the case for maintaining a highly accommodative stance of monetary policy.”

In the same year at Jackson Hole, the governor of the European Central Bank argued that demand side policies are justified as they, “help insure against the risk that a weak economy is contributing to hysteresis effects” (Draghi 2014). And in a recent Employment Outlook, the OECD argued that,

“Fiscal support during economic downturns . . . promotes labour market resilience by stabilising aggregate demand. It also reduces the risk of hysteresis, i.e. the risk that cyclical changes in unemployment or productivity as a result of the crisis persist even after aggregate demand has recovered.” (OECD 2017: 49).

Despite these acknowledgements of the risk of hysteresis by policy makers, most of the macroeconometric models used by policy making institutions do not allow for hysteresis effects. This is certainly true of the forecasting models used by central banks (Burgess et al 2013; Coenen et al 2018). More importantly, it is true of the models used by the European Commission and OECD to produce their influential country-specific estimates of the NAIRU (Havik et al 2014; Gianella et al 2008).

In this paper we introduce a hysteresis channel into the type of model used by the European Commission and OECD. In doing so, we ask whether or not the natural rate hypothesis holds in this type of model - or, in other words, whether or not the risk of hysteresis acknowledged by policy makers should be incorporated into the models used by policy institutions. We estimate the model using Bayesian methods for the three most populous European countries: Germany, France, and the United Kingdom. This approach allows us to evaluate the posterior probability that the natural rate hypothesis holds, which we find to be low in all three countries. This is a novel approach in the literature on
hysteresis, and suggests that the models used by the European Commission and OECD should be amended to reflect policy makers’ views on hysteresis.

The paper is structured as follows. Section 2 discusses the natural rate hypothesis and the modelling strategy used by the European Commission and OECD in more detail. Section 3 introduces our model and discusses its implications for unemployment and inflation under the hysteresis hypothesis. Section 4 discusses the estimation strategy and data, and section 5 presents the results. Finally, section 6 compares our results to the wider literature, and section 7 concludes.

2 The natural rate hypothesis

Since the seminal contributions of Phillips (1958), Samuelson and Solow (1960), Phelps (1967), and Friedman (1968), the vast majority of macroeconomists have acknowledged the existence of a trade-off between inflation and unemployment. Particularly, in response to temporary aggregate demand shocks, such as shocks to interest rates or monetary aggregates, shocks to government expenditure or taxation, or shocks to the financial system, inflation and unemployment are expected to move in opposite directions. This is usually formalised using the expectations augmented Phillips curve,

$$\pi_t = \pi_t^e + \beta(u_t - n_t) + \nu_t,$$

where $\pi_t$ denotes the inflation rate, $\pi_t^e$ denotes the expected inflation rate, $u_t$ denotes the unemployment rate, and $\nu_t$ denotes a white noise supply shock with $E[\nu_t] = 0$. The parameter $\beta$, which will usually be negative, determines the extent of the short run trade-off. To make the equation operational, one usually assumes that $\pi_t^e$ can be approximated by $\pi_t - 1$, which is the optimal forecast when agents believe inflation to follow a random walk. In this case we arrive at the simple accelerationist Phillips curve,

$$\Delta \pi_t = \beta(u_t - n_t) + \nu_t,$$

from which it follows that $E[\Delta \pi_t|u_t = n_t] = 0$ and $E[\Delta^2 \pi_t|u_t = n_t] = 0$, i.e. the inflation rate is stable when the unemployment rate is equal to $n_t$. Thus $n_t$ is the non-accelerating inflation rate of unemployment, or NAIRU, referred to in section 1.

The natural rate hypothesis states that $n_t$ is unaffected by aggregate demand shocks. However, as the increases in European unemployment rates observed during the 1980s were not accompanied by decelerating prices, the NAIRU cannot be a constant. A simple modelling framework to capture these requirements is the unobserved components model,

$$c_t = \phi_1 c_{t-1} + \phi_2 c_{t-2} + \zeta_t,$$

$$n_t = n_{t-1} + \eta_t,$$

$$u_t = n_t + c_t,$$

where $c_t$ is a stationary process representing the cyclical part of the unemployment rate. The white noise process $\zeta_t$ can be interpreted as an aggregate demand shock, as it moves $u_t$
and $\pi_t$ in opposite directions when $\beta < 0$, and the white noise process $\eta_t$ can be interpreted as shocks to those institutional features of the economy that drive the NAIRU. Note that $c_t$ follows an AR(2) process as this is the minimum number of lags needed for a cyclical process, and $n_t$ follows an AR(1) process as this is the minimum number of lags needed for a unit root process. The European Commission and OECD use this basic structure to produce their NAIRU estimates, with elaborations including the addition of exogenous variables in the Phillips curve, the use of wage inflation or real unit labour costs instead of price inflation, and the addition of time-varying intercepts in the NAIRU equation. We discuss these elaborations in the European Commission model in more detail in appendix B, and the reader can refer to Gianella et al. (2008) for a discussion of the OECD model. As the long run Phillips curve is vertical in the model sketched here, there is no long run trade-off between inflation and unemployment.

### 3 The hysteresis hypothesis

Various hysteresis mechanisms have been discussed in the theoretical literature. These include, but are not limited to, the insider-outsider mechanisms of Blanchard and Summers (1986, 1987, 1988), the skill-loss mechanism of Pissarides (1992), and the social norm mechanisms of Skott (2005) and Stockhammer (2011). Galí (2018) uses the insider-outsider approach to incorporate hysteresis into a New Keynesian model, and argues that unions set wages to target a weighted average of expected employment levels, where target employment is an exponentially weighted moving average of previous employment levels. This is similar to the Hargreaves-Heap (1980) specification,

$$ n_t = n_{t-1} + \alpha(u_{t-1} - n_{t-1}) + \eta_t, $$

which can be rearranged to yield,

$$ n_t = (1 - \alpha)n_{t-1} + \alpha u_{t-1} + \eta_t, $$

and therefore defines the NAIRU as an exponentially weighted moving average of previous unemployment rates. This is a popular specification in the relatively small literature that tests for hysteresis using unobserved components models (Jaeger and Parkinson 1994; Assarsson and Jansson 1998; Logeay and Tober 2006; Di Sanzo and Pérez-Alonso 2011), which our approach builds upon. It is particularly useful as the degree of hysteresis is represented by the scalar parameter $\alpha$, and when $\alpha = 0$ the equation of motion for the NAIRU is identical to that used in the simple natural rate model outlined in section 2. At the same time, we favour as simple an approach as possible when modelling European unemployment rates, as time series with a useful span are only available at annual frequency\(^1\).

Given the foregoing, the model used in the present paper is as follows:

$$ \Delta \pi_t = \beta c_t + v_t, \quad (1) $$

\(^1\)The United Kingdom, for example, only has quarterly unemployment rate data consistent with the LFS definition after 1992; all quarterly data prior to this is interpolated. As any meaningful study of the natural rate hypothesis has to include the 1970s and 1980s in its sample, we are therefore limited to the use of annual data.
\[ c_t = \phi_1 c_{t-1} + \phi_2 c_{t-2} + \zeta_t, \quad (2) \]
\[ n_t = n_{t-1} + \alpha c_{t-1} + \eta_t, \quad (3) \]
\[ u_t = n_t + c_t, \quad (4) \]

where the variable definitions are as before. This model uses the simple accelerationist Phillips curve and decomposes the unemployment rate into a stationary cyclical process and the NAIRU. However, in comparison to the European Commission and OECD approach, we allow the cyclical component of the unemployment rate to affect the NAIRU via the parameter \( \alpha \). We can therefore examine the exogenous NAIRU hypothesis empirically, formalised as \( H_0 : \alpha \approx 0 \), with minimal extraneous assumptions.

Although the hysteresis parameter \( \alpha \) is a reduced form parameter, its practical interpretation is quite intuitive, and can be understood by considering the reduced form Phillips curve implied by (1) - (4). To derive this, substitute \( c_{t-1} \) out of (3) using (4), and then substitute \( n_t \) out of the resulting expression using (1). This yields the reduced form Phillips curve,

\[ \Delta \pi_t = (1 - \alpha) \Delta \pi_{t-1} + \beta \Delta u_t + \xi_t, \quad (5) \]

where \( \xi_t = v_t - \beta \eta_t - (1 - \alpha)v_{t-1} \) is a reduced form MA(1) error term. Under the maintained hypothesis of no hysteresis, we have,

\[ \Delta^2 \pi_t = \beta \Delta u_t + \Delta v_t - \beta \eta_t, \]

which, following the notational convention used in Harvey (1990: 45), can be written as,

\[ \Delta \pi_t = \beta u_t + v_t - \beta \Delta^{-1} \eta_t, \quad (6) \]

where \( \Delta^{-1} \eta_t \) is a unit root process. Thus the reduced form Phillips curve implied by (1) - (4) when \( \alpha = 0 \) is the standard accelerationist Phillips curve augmented with a unit root intercept driven by the NAIRU shock process \( \eta_t \).

In contrast, and using the same notation, the hysteretic Phillips curve in (5) implies the long run relationship,

\[ \mathbb{E} [\pi_t | u_t, \xi_t] = \left( \frac{\beta}{\alpha} \right) u_t + \left( \frac{1}{\alpha} \right) \Delta^{-1} \xi_t. \quad (7) \]

When \( \alpha > 0 \), therefore, the model described by (1) - (4) implies the existence of a long run trade-off between inflation and unemployment. The position of the trade-off is affected by the supply and institutional shocks \( v_t \) and \( \eta_t \), but is independent of the aggregate demand shocks \( \zeta_t \). Thus the hysteresis hypothesis implies a considerably different relationship between unemployment and inflation than the natural rate hypothesis, allowing us to identify the degree of hysteresis.
4 Estimation

As discussed above, we use data at annual frequency to estimate the model described by (1) - (4), with the available sample spanning the period 1960 - 2017. This a relatively short time series on which to estimate unobserved components models, and for this reason we use Bayesian methods with informative priors. This is rapidly becoming the de facto estimation approach in macroeconometrics, and can be expected to improve the frequency properties of the estimator. At the same time, the use of Bayesian estimation allows us to avoid placing dogmatic priors on the “signal to noise ratio” (i.e. the variance ratios of one or more of \( v_t, \zeta_t, \) and \( \eta_t \)), which is common practice in frequentist approaches to the estimation of unobserved components models (Stock 1994; Staiger et al 1997; Laubach 2001; Llaudes 2005).

The use of Bayesian methods is made easier by the existence of a fairly obvious prior for the main parameter of interest, \( \alpha \). This parameter represents the hysteresis effect, and we therefore have strong theoretical reasons to expect a non-negative value. At the same time, following Blanchard (2018) and others, we should want to follow a conservative estimation procedure by placing a lot of weight on the null hypothesis of no hysteresis. These two features of our prior information are well described by the exponential distribution, which is the maximum entropy distribution with given mean and support on \([0, \infty)\), and has mode equal to zero (i.e. no hysteresis). In the main estimations we use an exponential prior on \( \alpha \) with mean and standard deviation equal to 0.3. This results in the prior distribution illustrated in figure 1, which has approximately 28% of the density lying in \([0, 0.1]\), and approximately 49% of the density lying in \([0, 0.2]\). As this prior places a lot of weight on the null hypothesis \( H_0: \alpha \approx 0 \), it is relatively conservative.

Stationarity in the cyclical component of the unemployment rate in (2) implies the existence of complex conjugate eigenvalues with positive real part and modulus less than unity. As pointed out in Planas et al (2008), it is difficult to place priors that incorporate this information on the parameters in (2), as the implied restrictions define a relatively complicated subset of the parameter space (see Hamilton 1994: 17). However, if we re-parameterise (2) as,

\[
c_t = 2A\omega c_{t-1} - A^2 c_{t-2} + \zeta_t,
\]

then the eigenvalues can be written as,

\[
A\omega \pm A\sqrt{\omega^2 - 1},
\]

which implies the restrictions \( \omega^2 < 1 \) (complex conjugate), \( A\omega > 0 \) (positive real part), and \( A^2 < 1 \) (modulus less than unity). To impose these restrictions without loss of generality, we can restrict the \((A, \omega)\) parameter space to the unit square, which implies a uniform prior for \( A \) and \( \omega \) as the maximum entropy distribution for a finite interval.

For the \( \beta \) parameter in (1), although we would usually expect this to be negative, it is not necessarily implied by the structure of the model. As a result, we use a standard normal

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\(^2\)See An and Schorfheide (2007) for a discussion of the use of Bayesian methods to estimate DSGE models, and Gelman et al (2004) for a discussion of the use of Bayesian techniques to improve the frequency properties of estimators. Intuitively, the penalisation of the likelihood function by a prior should reduce inefficiency at the (potential) expense of increased bias; ideally this reduces the mean squared error of the estimator.
Figure 1: Prior densities for $\alpha$, $\beta$, $A$, $\omega$, and the shock standard deviations.

prior for $\beta$. This is relatively tight, although as $\Delta\pi_t$ and $c_t$ will generally be of the same order of magnitude we do not expect $|\beta|$ to be much greater than 2. Finally, we use inverse gamma priors for the standard deviations of $v_t$, $\zeta_t$, and $\eta_t$, which is fairly common in the literature (e.g. Gelman et al 2004: 50), and set the prior means to 1.5 for $v_t$ and 0.75 for $\zeta_t$ and $\eta_t$, which are consistent with the sample standard deviations of $\Delta\pi_t$ and $\Delta u_t$. The standard deviations for the inverse gamma distributions are equal to $\infty$, resulting in rather loose priors. The full set of prior distributions is plotted in figure 1. The priors are relaxed along several dimensions in section 5.2, where we consider the robustness of our results to the choice of prior. Given the priors, the models are estimated with a diffuse Kalman filter and MCMC integration using Dynare, which is described in more detail in appendix C.

The most reliable source of annual unemployment and inflation series for Germany, France, and the UK, is the AMECO database, which is used to estimate the European Commission NAIRU models discussed in section 2 and appendix B. We use these data, specifically the unemployment rate coded ZUTN, and the consumer price index coded ZCPIN. As the unemployment rate is recorded in percentage points (i.e. 1% unemployment is recorded as “1”, rather than “0.01”), we define the inflation rate as $100 \times \Delta P_t / P_{t-1}$, where $P_t$ is the price index at time $t$. The sample runs from 1960 to 2017 for all countries, aside from the unemployment rate data for Germany which is split between West Germany (prior to unification) and Germany (after unification). Some back-casting was therefore performed, which is described in detail in appendix A.

Figure 2 plots the unemployment and inflation rate data for the three countries. Inflation rates were high in the 1970s, with Germany experiencing a relatively modest increase, and
Figure 2: Time series plots of annual unemployment rates (top panel) and inflation rates (bottom panel) for Germany, France, and the UK.
then decreased for all countries during the Great Moderation. In contrast, unemployment rates were relatively low during the 1960s and 1970s, and then increased rapidly in all countries during the 1980s. Unemployment remains elevated in France at the time of writing, and is relatively low in the UK and Germany. Figure 3 plots kernel density estimates for $\Delta u_t$ and $\Delta \pi_t$ for each country. Although most of the variables are leptokurtic, they are not significantly skewed, and we can therefore be confident that the linear model described in section 3 will provide a reasonable approximation to the joint distribution.

5 Results

5.1 Parameter estimates

Table 1 summarises the posterior distributions for the three countries under investigation, with graphical comparisons of the prior and posterior distributions presented in figures D.1 to D.3 in appendix D. The estimates of $\alpha$ are rather imprecise, particularly for France and the UK. With this in mind, the posterior means for $\alpha$ are greater than the prior mean of 0.3, implying that the data suggest an upwards revision to our maintained hypothesis $H_0 : \alpha \approx 0$. In addition, it is worth noting that our estimates are in line with those found in Assarsson and Jansson (1998) and Di Sanzo and Pérez-Alonso (2011), who estimate models
Table 1: Parameter estimates

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<th>Germany</th>
<th>France</th>
<th>UK</th>
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<tbody>
<tr>
<td></td>
<td>prior mean</td>
<td>posterior mean</td>
<td>90% highest</td>
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<tr>
<td></td>
<td></td>
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<td>posterior density</td>
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<tr>
<td>$\alpha$</td>
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</tr>
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<td>$\beta$</td>
<td>0.00</td>
<td>-0.67</td>
<td>[-1.33 0.00]</td>
</tr>
<tr>
<td>$A$</td>
<td>0.50</td>
<td>0.76</td>
<td>[0.42 0.86]</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.50</td>
<td>0.76</td>
<td>[0.58 1.00]</td>
</tr>
</tbody>
</table>

with a similar structure to our own (albeit with Okun’s law in place of the Phillips curve). The country with the lowest hysteresis parameter in our sample is Germany, which may be driven by its subdued inflationary experience in the 1970s. This is discussed in Issing (2005), who suggests that an early adoption of strict monetary targeting by the Bundesbank, and a fear of inflationary spirals among union wage setters, kept inflation under control during this period.

Of more interest than the posterior means are the posterior probabilities reported in table 2. This presents the probabilities $\Pr[\alpha < \epsilon]$ for $\epsilon = \{0.05, 0.1, 0.2\}$ implied by the posterior distributions for each country, which in most cases are very small. Thus, for example, our estimates imply that the probabilities that $\alpha$ is less than 0.1 are 3.4% for Germany, 1.1% for France, and 0.9% for the UK. Given these results, and despite the evident difficulty in pinning down a precise value for the degree of hysteresis, we can confidently state that the data reject the natural rate hypothesis. As such, we can conclude that hysteresis effects should be incorporated into the NAIRU models used by the European Commission and OECD, supporting the quoted acknowledgements of hysteresis risks voiced by policy makers in section 1.

The remaining posterior means are sensible, with negative values for $\beta$, and values of $\omega$ implying cycle lengths of 9.5, 8.7, and 8.9 years for Germany, France, and the UK, respectively. The posterior distributions of the shock standard deviations are also sensible, with no evidence of the pile-up problem. Using the posterior means of $\alpha$ and $\beta$, the long run trade-off implied by equation (7) in section 3 is approximately equal to unity for each country. As a result, aggregate demand management policy should be able to reduce the
long run unemployment rate by approximately one percentage point in Germany, France, and the UK, at the expense of a one percentage point increase in the long run inflation rate. Although a detailed discussion of the policy implications of hysteresis is beyond the scope of this paper, it is worth noting that this is a reasonable trade-off in the case of France, where the annual inflation rate was below 2% from 2011 to 2017, and the unemployment rate was above 9% over the same period.

Finally, the implications for the NAIRU of a negative aggregate demand shock are illustrated in figure 4. This plots the impulse response functions to a one percentage point increase in the observed unemployment rate resulting from a shock to $\zeta_t$. After 20 years, our estimates imply that the expected unemployment rate is approximately 1.2 percentage points higher than its starting value in Germany, 1.54 percentage points higher in France, and 1.52 percentage points higher in the UK. The corresponding minima of the 90% highest posterior density regions are above 0.5 percentage points for the UK and France, and above zero for Germany. These reinforce the estimates in tables 1 and 2, again implying that the natural rate hypothesis should be rejected in favour of the hysteresis hypothesis in Germany, France, and the UK.

5.2 Alternative priors

The main drawback to Bayesian econometrics is the dependence of the final estimates on the choice of prior. As such, we now consider the robustness of the results discussed in section 5.1 to an alternative prior. Specifically, the prior on $\alpha$ is set to a standard normal distribution, the prior on $\beta$ is maintained as standard normal, and we use the common specification of an AR(2) process for the cyclical unemployment rate, i.e. (2), where the priors on $\phi_1$ and $\phi_2$ are again standard normal. Finally, we retain the same priors on the shock standard deviations as in section 5.1. Thus $\alpha$ is no longer constrained to be positive, and is wider than the exponential prior used in our baseline results, and the priors on the cyclical component of the unemployment rate are looser.

Table 3 summarises the posterior distributions for the three countries, with graphical comparisons of the prior and posterior distributions presented in appendix E. Again, the estimates of $\alpha$ are rather imprecise, but in all cases the prior means are greater than those in our baseline results. This is not particularly surprising, as the standard normal prior is less conservative than the exponential prior. The posterior probabilities $\Pr[\alpha < \epsilon]$ presented in table 4 are comparable to the corresponding estimates in table 2, despite non-zero mass over $\mathbb{R}^-$, with the notable exception of the UK. This reflects the bi-modal posterior distribution in figure E.6. As the posterior probabilities in table 4 for Germany and France remain small, and $\Pr[\alpha < \epsilon]$ is still only around 20% for $\epsilon = \{0.05, 0.1, 0.2\}$ in the UK, we can reasonably retain our conclusion that the data reject the natural rate hypothesis.

<table>
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<th>Posterior probability</th>
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<th>UK</th>
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<tr>
<td>$\Pr[\alpha &lt; 0.05]$</td>
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<td>0.011</td>
<td>0.009</td>
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<tr>
<td>$\Pr[\alpha &lt; 0.1]$</td>
<td>0.066</td>
<td>0.022</td>
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<tr>
<td>$\Pr[\alpha &lt; 0.2]$</td>
<td>0.149</td>
<td>0.062</td>
<td>0.051</td>
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</table>

Table 2: Posterior probabilities
6 Relation to the existing literature

The results presented in sections 5.1 and 5.2 suggest that the natural rate hypothesis should be rejected in favour of the hysteresis hypothesis, and hysteresis effects should be incorporated into the NAIRU models used by policy making institutions. As discussed in section 3, this result builds on the relatively small literature using unobserved components models to estimate hysteresis effects, which includes Jaeger and Parkinson (1994), Assarsson and Jansson (1998), Logeay and Tober (2006), and Di Sanzo and Pérez-Alonso (2011). This literature generally finds evidence of hysteresis in European labour markets. With the exception of Logeay and Tober (2006), however, the existing studies in this literature use Okun’s Law in place of the Phillips curve, and as a result do not directly impinge upon the natural rate hypothesis. To the best of our knowledge, the model used in the present paper is the first to introduce a simple hysteresis channel into the NAIRU model used by the European Commission and OECD, and the first to estimate the model using Bayesian methods.

Our results support a growing empirical literature on the existence of hysteresis. Traditionally this literature has utilised unit root tests, with the existence of a unit root in the unemployment rate being seen as evidence of hysteresis (see Stanley 2004 for a useful survey). However, as discussed above, the behaviour of unemployment and inflation in Europe during the 1980s makes a constant NAIRU all but impossible, and this is reflected in the time varying NAIRU models used by the European Commission and OECD. As the
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<td>$\beta$</td>
<td>0.00</td>
<td>-0.53</td>
<td>[-1.61 0.66]</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>0.00</td>
<td>0.81</td>
<td>[0.29 1.34]</td>
</tr>
<tr>
<td>$\phi_2$</td>
<td>0.00</td>
<td>-0.37</td>
<td>[-0.69 -0.04]</td>
</tr>
</tbody>
</table>

Table 3: Parameter estimates, alternative prior

unemployment rate in these models has a unit root by construction, the unit root approach to hysteresis is of limited utility. Of more interest is the reduced form Phillips curve approach, originally introduced in Gordon (1989), and applied in Fortin (1991) and Burger and Marinkov (2006). Gordon (1989) does not find evidence of full hysteresis, but finds “traces of hysteresis” in the USA and UK, where prolonged slumps are found to exert very little downward pressure on prices, and Fortin (1991) finds hysteresis effects in Canada after 1972. Burger and Marinkov (2006) applies the approach to South Africa, finding some evidence of hysteresis in output.

An alternative approach to hysteresis is to ignore the reduced form evidence between inflation and unemployment, and directly examine the impact of long term unemployment on wages or future job prospects. Arulampalam et al (2000), for example, use the British Household Panel Survey to demonstrate that British individuals’ past employment states directly affect their future labour market fortunes, “perhaps because of depreciation of human capital, or because employers use an individual’s previous labour market history as a screening mechanism.” Elsby et al (2015) provide a useful discussion of this issue in the context of the Beveridge curve, and Mathy (2018) examines duration effects in the US Beveridge curve during the Great Depression. Of particular interest are Kroft et al (2013), Eriksson and Rooth (2014), and Farber et al (2016), which examine the existence of duration dependence by sending fictitious job applications to US employers. Kroft et al (2013) find that the probability of receiving an interview decreases with the length of an applicant’s unemployment spell, and suggest that this is due to employer screening. Eriksson and Rooth (2014) find that past spells of unemployment are irrelevant to the probability of receiving
Table 4: Posterior probabilities, alternative prior

<table>
<thead>
<tr>
<th>Posterior probability</th>
<th>Germany</th>
<th>France</th>
<th>UK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pr[α &lt; 0.05]</td>
<td>0.074</td>
<td>0.019</td>
<td>0.213</td>
</tr>
<tr>
<td>Pr[α &lt; 0.1]</td>
<td>0.081</td>
<td>0.021</td>
<td>0.215</td>
</tr>
<tr>
<td>Pr[α &lt; 0.2]</td>
<td>0.105</td>
<td>0.028</td>
<td>0.218</td>
</tr>
</tbody>
</table>

an interview, but contemporary unemployment spells lower the probability of receiving an interview by twenty percent for low and medium skilled jobs. Farber et al (2016), in contrast, find that there is no relationship between callback rates and the duration of unemployment for college educated females.

It is also possible to examine the reduced form relationship between inflation and unemployment and duration effects simultaneously. Llaudes (2005) and Rusticelli (2014) estimate versions of the OECD NAIRU model that incorporate measures of long term unemployment as explanatory variables in the Phillips curve and NAIRU equations. Llaudes (2005) concludes that, “the incidence of long-term unemployment is key to understanding the true pressures on prices”, with high long term unemployment having little effect on inflation according to his estimates (ibid.: 20). Rusticelli (2014) finds similar results for Greece, Ireland, Italy, Portugal, and Spain, but finds statistically insignificant effects for Germany, France, and the UK. Her results do not rule out hysteresis effects in Germany, France, and the UK, but suggest that the methodology applied in the present paper - i.e. incorporating actual unemployment in the NAIRU equation - may be superior to the use of long term unemployment.

Finally, it is possible to examine hysteresis effects using VAR models, as in the recent paper by Rodriguez-Gil (2017), and cross-country panel regressions, as in Baccaro and Rei (2007), Stockhammer and Klär (2011), and Stockhammer et al (2014). These papers control for labour market institutions including employment protection, social security levels, union density, and so on, and find an important independent role of capital accumulation and real interest rates in driving medium run unemployment rates. Rodriguez-Gil (2017) finds evidence of various hysteresis channels alongside orthodox institutional mechanisms, particularly the generosity of unemployment benefits, in the UK and the Netherlands.

To summarise, the existing literature does not unequivocally support the hysteresis hypothesis, and where it does, it is not always evaluated against the natural rate hypothesis as defined in the present paper. Nevertheless, it is fair to interpret the literature as broadly supportive of the existence of hysteresis. Of particular importance are the two experimental papers on the existence of duration effects in the job market, Kroft et al (2013) and Eriksson and Rooth (2014), which find direct evidence for the microeconomic mechanisms underpinning the type of aggregate relationship explored in the present paper. Our results lend support to the existing literature, and are consistent with the existing evidence in favour of the hysteresis hypothesis.

7 Concluding remarks

A number of important policy makers have acknowledged the risk of unemployment hysteresis since the financial crisis of 2008. Nevertheless, hysteresis effects are not - as of yet -
incorporated into the models used by key policy making institutions. This is not without its risks. As recently demonstrated in Galí (2018), optimal monetary policy is highly responsive to changes in the unemployment rate in the presence of hysteresis, and fully stabilises the unemployment rate in response to aggregate demand shocks. Similarly, Engler and Ter- vala (2018) analyse fiscal policy in a model where hysteresis arises from learning by doing, and demonstrate that output and welfare multipliers are considerably higher than the mainstream consensus suggests. Their results support Stockhammer (2011) and Ball (2014), who argue that fiscal policy can and should be used as an ordinary tool of demand management in the presence of hysteresis. Currently, these channels are not reflected in models that produce influential macroeconomic forecasts and/or estimates of country-specific NAIRUs.

In this paper, we have incorporated a hysteresis channel into the type of NAIRU model used by the European Commission and OECD. By estimating the model using Bayesian methods, we have demonstrated that the posterior probability that the natural rate hypothesis holds is low for Germany, France, and the UK. In particular, a one percentage point demand shock to the unemployment rate results in a long run increase of 1.2 percentage points in Germany, 1.54 percentage points in France, and 1.52 percentage points in the UK. We use a novel approach to derive these results, which contribute to the existing empirical literature. As our results support the hysteresis hypothesis, we conclude that the models used in policy making institutions should be amended to reflect the existence of hysteresis effects, and that hysteresis effects can be captured in a satisfactory manner in this type of model (Lendvai et al 2015). Actual unemployment rates and estimated NAIRUs across the European Union remain high (European Commission 2019), suggesting the existence of substantial welfare losses. This need not be the case, as our results support the considerably more active role for aggregate demand management policy proposed by Galí (2018) and others, and suggest that long term unemployment can be permanently reduced.
References


A Data preparation for Germany

A number of the AMECO series for Germany begin after re-unification, with West German data provided prior to this. To create unified series for Germany, therefore, the German data has to be back-cast. Suppose a West German series \( \{x_t\} \) runs from \( t = t_0 \) to \( t = t_m \), and an equivalent German series \( \{y_t\} \) runs from \( t = t_m \) to \( t = t_n \). Denote by \( \delta_t \) the West German series growth rate from \( t - 1 \) to \( t \), i.e.,

\[
\delta_t = \frac{x_t - x_{t-1}}{x_{t-1}}. \tag{A.1}
\]

Prior to \( t_m \) - when German data is not observable but \( \delta_t \) is observable - we assume that the following relation holds:

\[
y_t = (1 + \delta_t)y_{t-1}. \tag{A.2}
\]

Prior to \( t_m \), we can therefore back-cast the German series by recursively calculating,

\[
y_{t-1} = \frac{y_t}{1 + \delta_t}. \tag{A.3}
\]

This is a fairly common approach to back-casting, and is used (for example) by the Bank of England in its “millennium of macroeconomic data” dataset. A comparison of the amalgamated unemployment rate used in the present paper with the corresponding West German series is shown below:
B The European Commission NAIRU approach

The European Commission’s approach to estimating the NAIRU is a common one, both in the academic literature and international organisations including the OECD - see Gianella et al (2008). Note, however, that they refer to the series as a NAWRU, or “non-accelerating wage rate of unemployment”, as wage inflation or real unit labour costs rather than price inflation are used in the Phillips curve. The European Commission approach involves an unobserved components model of the following form,

\[ n_t = \mu_{t-1} + n_{t-1} + \eta_t, \]
\[ \mu_t = \mu_{t-1} + \varepsilon_t, \]
\[ c_t = \phi_1 c_{t-1} + \phi_2 c_{t-2} + \zeta_t, \]
\[ u_t = n_t + c_t, \]
\[ w_t = f(c_t, z_t), \]

where the notation is consistent with the main body of the paper, and \( \mu \) is a time varying drift term in the NAIRU. The error terms are mutually uncorrelated white noise processes, and if the variance of \( \varepsilon_t \) is small the NAIRU will behave in a similar manner to an integrated random walk. The model is completed with a Phillips curve in \( w_t \), which is either the second difference of log wages or the first difference of log real unit labour costs, which is a function of the cyclical unemployment rate and a vector \( z_t \) of control variables. The choice of dependent variable in the Phillips curve, and the rationale behind it, is detailed in Havik et al (2014). Havik et al report that the use of real unit labour costs results in slightly less volatile NAIRU estimates, but for most countries the difference between the two methods appears to be minor.

Alongside the differences in the dependent variable, those country models that are estimated with a traditional Keynesian Phillips curve use different specifications for the exogenous regressors, including the terms of trade, labour productivity, and various transformations of the labour share of income. General ARMA errors can also be incorporated, and upper and lower bounds on the parameter estimates are imposed on the estimation procedure. As at least some of these constraints are usually binding, this implies the imposition of signal-to-noise ratios in the European Commission models, which can be interpreted as dogmatic priors. The interested reader can refer to the excel specification files available at https://circabc.europa.eu. These estimates are relatively straightforward to reproduce with Stata or Matlab.
C Bayesian estimation

The general form of a state space model can be written as,

\[ \gamma_t = F \gamma_{t-1} + \nu_t, \]  
(C.1)

\[ y_t = Ax_t + H \gamma_t + w_t, \]  
(C.2)

where \( \gamma_t \) is an \((r \times 1)\) vector of unobserved state variables, \( F \) is an \((r \times r)\) matrix of parameters, \( \nu_t \) is an \((r \times 1)\) vector of normally distributed innovations, \( y_t \) is an \((n \times 1)\) vector of observed dependent variables, \( x_t \) is a \((k \times 1)\) vector of observed independent variables, \( A \) is an \((n \times k)\) matrix of parameters, \( H \) is an \((n \times r)\) matrix of parameters, and \( w_t \) is an \((n \times 1)\) vector of normally distributed innovations which are independent of \( \nu_t \). If we collect the observations through to \( t - 1 \) in the vector \( z_{t-1} = (y_{t-1}, y_{t-2}, ..., y_1, x_{t-1}, x_{t-2}, ..., x_1) \), then we have,

\[ y_t | x_t, z_{t-1}; \theta \sim N(\mu_t, \Sigma_t), \]

where \( \mu_t \) and \( \Sigma_t \) are functions of \( x_t, z_{t-1} \), and the vector \( \theta \), which contains the unknown parameters in \( F, A, H \), and the variance-covariance matrices of \( \nu_t \) and \( w_t \). These can be evaluated numerically using the Kalman filter (see e.g. Hamilton 1994), and the value of the log-likelihood function evaluated at \( \theta \) is,

\[ \log L(\theta | y_t, x_t, z_{t-1}) = \sum_{t=1}^{T} \log p(y_t | x_t, z_{t-1}; \theta) = 
- \frac{Tn}{2} \log(2\pi) - \frac{1}{2} \sum_{t=1}^{T} \log(|\Sigma_t|) - \frac{1}{2} \sum_{t=1}^{T} (y_t - \mu_t)' \Sigma_t^{-1} (y_t - \mu_t), \]  
(C.3)

which reflects the probability of observing the sample if \( \theta \) were the true value and (C.1) - (C.2) the true model (Hamilton 1994).

Using Bayes theorem, we know that the posterior distribution of \( \theta \) is proportional to the likelihood function (C.3) multiplied by our prior distribution, i.e.,

\[ p(\theta | y_t, x_t, z_{t-1}) \propto L(\theta | y_t, x_t, z_{t-1}) p(\theta). \]  
(C.4)

Bayesian estimation in Dynare then proceeds as follows (Griffoli 2013):

1. Using the Kalman filter - in this case diffuse - to evaluate the likelihood in (C.3), locate the posterior mode given (C.4) using a numerical maximisation procedure.

2. Using a numerical integration algorithm - in this case Markov Chain Monte Carlo - approximate the posterior distribution around its mode located in step 1.

We use 3 parallel chains in the Markov Chain Monte Carlo algorithm, with 2 million replications per chain, from which we discard the first 50% as a burn-in. The initial proposal distribution is multivariate Student’s \( t \) with 3 degrees of freedom, and we set the “scale” parameter to achieve an acceptance rate of between 25% and 33%. This leads to apparently acceptable convergence diagnostics.
D  Baseline priors and posteriors

Figure D.1: Prior and posterior distributions, German data.
Figure D.2: Prior and posterior distributions, French data.
Figure D.3: Prior and posterior distributions, UK data.
E Alternative priors and posteriors

Figure E.4: Prior and posterior distributions with alternative prior, German data.
Figure E.5: Prior and posterior distributions with alternative prior, French data.
Figure E.6: Prior and posterior distributions with alternative prior, UK data.