

Worker debt, default and diversity of financial fragility

Matthieu Charpe
Peter Flaschel

Worker debt, default and diversity of financial fragility

Matthieu Charpe*

International Labor Organization
Geneva, Switzerland

Peter Flaschel

Bielefeld University
Bielefeld, Germany

April 11, 2011

Abstract

This paper presents a model addressing the conditions under which financial instability arises in the event of household debt. The model addresses two main cases. First, household debt is affected by functional income distribution. Second, household debt is affected by credit supply and depends on bank performances. The model shows that financial fragility arises through a Fisher effect in the first case and through a debt financed consumption boom in the second case. The model then explores two extensions. First, we raise the question of debt default and its impact on financial instability. Second, we discuss the ability of capital adequacy ratio to limit financial instability.

Keywords: Households debt, booms, commercial banks, credit rationing, Minsky.

JEL CLASSIFICATION SYSTEM: E24, E31, E32.

1 Introduction

Household debt has played a key role in the current financial crisis. There are two main views accounting for the over-indebtedness of households in the literature. The first view focuses on demand side explanations. Greater borrowing is linked to higher demand for credit from households in the face of a deterioration of labour income. This explanation stresses the key role played by both functional and personal income distributions.¹ The decline in the wage share coupled with a more unequal distribution of labour income

*Corresponding author: charpe@ilo.org

¹See for instance Barba and Pivetti (2009) and Pollin (1988).

between top and lower deciles of the distribution has forced low income households to substitute wage increases for debt. The demand side view also focuses on conspicuous consumption la Veblen, where low income households rely on credit to gain access to luxury goods consumed by the elites.

The second view focuses on supply side explanations. Financial institutions have increasingly supplied credit to households in general, and to low income households in particular. This approach stresses the role of financial innovation such as securitization in reducing perceived credit risk. Financial deregulation and increased competition between financial institutions lead to less vigilant monitoring of borrowers. Home equity lending, where real estate is used as collateral for credit supply, is also one factor explaining the large flows of credit that have been channeled towards households.²

This paper presents a macroeconomic model to analyse the main features and properties of these two explanations of household debt. The aim is also to understand the specific drivers of financial instability associated with these two theoretical approaches. The purpose is not to oppose both explanations, as it is likely that both demand and supply side elements have contributed to household over-indebtedness. It is, however, useful to discuss their similarities and differences by making use of a macroeconomic model.

The model consists of three elements: household debt, income distribution and a banking sector. The model is constructed so that setting some parameters at zero enables us to consider the case with or without income distribution as well as the case with or without credit rationing. Household debt interacts in one case with income distribution and in the other case with the banking sector.

The model has the following characteristics. The demand side explanation is captured by focusing on functional income distribution rather than personal income distribution.³ Workers raise debt to finance the gap between their income and their consumption. Income distribution affects worker labour income and their consumption decisions. The model is based on the framework developed in Charpe et al. (2009) and Charpe et al. (2011). The supply side explanation is captured by credit rationing, with worker consumption being a positive function of credit supplied by banks. Credit supply is usually expressed as a function of borrower characteristics.⁴ Here, however, it is a function of bank characteristics, with banks granting loans on the basis of their own performance.

In the case of income distribution and household debt, we show that debt accumulation

²See for instance Crotty (2009), Wray (2009) or Palma (2009).

³Functional income distribution is modeled following Proaño et al. (2010) and Chiarella et al. (2005).

⁴See Dutt (2006) for instance.

in nominal terms is stable due to the recessionary effect of income transfers from workers-borrowers with a high propensity to consume to capitalists-lenders with a low propensity to spend. Despite the overall stability of the system, there is a tendency for household debt to produce price deflation and a cumulative dynamic of debt in real terms similar to Fisher debt-deflation spirals. We also show that the traditional keynesian consumption function has difficulties reproducing the substitution of wage increases for debt stressed by demand side explanations. The Keynesian consumption function implies a positive correlation between household income and debt. This first case calls for the need of an alternative consumption function to support the demand side explanation (see Barba and Pivetti (2009) for a few possible alternatives). In the case of credit rationing, we show that the dynamic of debt is unstable. Credit rationing generates debt-financed consumption booms. Aggregate demand expands with consumption financed by banks credit.

We then discuss two important extensions: debt default and prudential regulation. Debt default is a key feature of the current financial crisis. Over-indebtedness leads households to default on debt. The boomerang effect of default on banks through non-performing loans can be seen as reason for the duration of the crisis. We show that default stabilizes debt accumulation in the case of demand side explanation. Default reduces the income transfers from workers-borrowers with a high propensity to consume to capitalists-lenders with a low propensity to spend. Default, on the contrary, generates a credit crunch in the event of credit rationing.

Lastly, the crisis has shed light on the limits of prudential regulation. We therefore assess the case where banks adjust the mark-up on household debt to meet a prudential capital adequacy ratio. We show that such prudential ratios have pro-cyclical effects. The mark-up affects income transfers between borrowers and lenders as well as the income of workers. Prudential ratio is, however, stabilizing in the case of credit rationing due to its impact on bank profitability.

This paper is structured as follow. Section 2 presents the main equations and assumptions of the model. Section 3 discusses the reduced form equations and steady states. Sections 4 outlines the case of income distribution and debt, while section 5 addresses the case of credit rationing and debt. Section 6 explores debt default, and Section 7 prudential ratio. Different tables, graphics and numerical simulations as well as the proofs of the stability conditions are contained in the appendix.

2 Household debt, income distribution and credit rationing

This section presents the different agents forming part of the economy as well as the main accounting and behavioral equations. There are two types of households in the economy. Worker household receives labour income and finances the discrepancy between their income and consumption by raising new debt. Capitalist household owns financial assets and saves all its incomes. Assuming two households with different propensities to consume gives a central role to income distribution between labour and capital as well as income distribution between borrowers and lenders.

Worker nominal income Y_w is equal to labour income wL^d minus interest payments on debt $i_\lambda\Lambda$, with w wages, L^d labour demand, i_λ the interest rate on debt and Λ the debt. Workers consume an homogenous goods at a price p . Workers consume all their net income plus the quantity of credit supplied by firms $\dot{\Lambda}$.⁵ It follows that the propensity of workers to consume is larger than one. This ensures a positive debt to capital ratio at the steady state.⁶ At each period, households default on debt at a rate φ_λ . The rate of debt default is endogenous and increases at a rate β_φ when household income Y_w drops below the income's steady state Y_{w0} .

This model does not explicitly address the issue of housing debt, which would require specifying two goods: consumption goods and housing goods. Modeling a housing sector with capital accumulation, profitability and prices would make the model overly complicated in light of the mechanisms we are interested in exploring here.

Workers' debt:

$$Y_w = wL^d - i_\lambda\Lambda \quad (1)$$

$$pC_w = Y_w + \dot{\Lambda} \quad (2)$$

$$\varphi_\lambda = \beta_\varphi(Y_{w0} - Y_w) + n_\varphi \quad (3)$$

Table 1 presents bank balance sheet. Banks collect deposits D and supply credits Λ to households. Profits Π_b consist of interests on loans granted to households, minus interest i_d paid on deposits. A share α_{π_b} of profits is distributed to asset holders who own banks. Profits are defined as profits before losses on loans, such that losses are supported by banks and are not transferred to capitalists. This assumption simplifies the structure of the model and makes the stability conditions analytically tractable. Profits are positive

⁵We define $\dot{X}$ as the change of variable X , and $\hat{X}$ as the growth rate of variable X .

⁶The total propensity to consume in the economy is, however, lower than one as capitalists save all their income.

at the steady state. This entails that the net wealth of banks W_b^n is positive. Bank net wealth increases with retained earnings and decreases with debt default (see Eq 6).

Commercial banks:

$$\Pi_b = i_\lambda \Lambda - i_d D \quad (4)$$

$$W_b^n = \Lambda - D \quad (5)$$

$$\dot{W}_b^n = (1 - \alpha_{\pi_b})(i_\lambda \Lambda - i_d D) - \varphi_\lambda \Lambda \quad (6)$$

$$\dot{\Lambda} = cY_w - \varphi_\lambda \Lambda \quad (7)$$

$$c = c_e + \beta_c \left[W_b^n - W_{b0}^n \right] \quad (8)$$

$$i_d = cst \quad (9)$$

$$i_\lambda = i_d + \xi_\lambda + \beta_{bis} \left[\left(\frac{W_b^n}{\Lambda} \right)_0 - \frac{W_b^n}{\Lambda} \right] \quad (10)$$

The stock of debt increases with new credit and decreases with debt default (see Eq 7). Multiplying the rate of default and the existing stock of debt $\varphi_\lambda \Lambda$ gives the quantity of debt default at every period. New credit is a non-linear function of worker net income cY_w . Credit rationing is modeled following Duménil and Levy (1999).⁷ Credit supply depends on two components, which capture the heterogeneity of borrowers with respect to credit rationing. Some borrowers are not subject to credit rationing. They demand a quantity of credit $c_e Y_w$ to cover the fraction of their consumption in excess of their income. Some borrowers are subject to credit rationing. Their consumption in excess of their income is determined by the credit supply of banks. This paper assumes that the credit decisions of banks depend on their performances. Banks relax credit rationing and expand credit supply to workers, when banks net wealth is above its long run value W_{b0}^n . Banks are less vigilant and reduce the monitoring of debt in periods of expansion. Total consumption (Eq 2) is now a function of both worker income and bank credit policy.

As in Barbosa-Filho and Taylor (2006); Dos Santos and Zezza (2005), there is no central bank, in order to keep the model simple. Commercial banks create money endogenously and meet their need for funds a posteriori through an increase in deposits. There is no compulsory reserves in this setting as it would require the introduction of a central bank and an increase in the number of dynamic equations. This also entails that there is no Taylor rule. The interest rate on deposits i_d is therefore assumed to be constant.

The mark up, however, is endogenous and adjusts to meet a targeted capital adequacy ratio, e.g. as set by the Bank for International Settlement, and to cover loans losses (Eq 10).

⁷Although Dumenil and Levy address the question of firms' debt.

Capital adequacy ratio is the ratio between banks' own funds, or banks' net wealth, and the stock of risky assets (the stock of debt). The interest rate increases at a speed β_{bis} when the capital adequacy ratio $\frac{W_b^n}{\Lambda}$ drops below its long run level $\left(\frac{W_b^n}{\Lambda}\right)_0$ in an attempt to restore profitability⁸. This follows Lavoie and Godley (2007), as well as Le Heron and Mouakil (2008). The performance of banks affects the real economy through two channels: credit supply to workers and the interest rate on debt.

The dynamic of the economy in the presence of debt default and pro-active banks is ambivalent. On the one hand, debt default stabilizes the accumulation of debt (Eq 7). The strength of this stabilizing feedback channel is given by the parameter β_φ , which measures the sensitivity of debt default to changes in income. On the other hand, debt default reduces bank performance and leads to a tightening of credit as well as an increase in interest rates. These two mechanisms are governed by the elasticities of credit supply β_c and interest rates β_{bis} to the performances of banks (see Eq 8 and 10).

The second set of households are capitalists, who own firms and banks. Their income Y_c is made of the totality of firm profits rpK , a share of bank profits $\alpha_{\pi_b}\Pi_b$ and deposit interests i_dD . Capitalists save all their income. The model does not consider the case of positive capitalist consumption. However this would not change the main properties of the model, as long as the propensity to consume of capitalists remains lower than that of workers. A redistribution of income from labour to capital reduces the overall propensity to spend in the economy. The investment of firms is financed out of the income of asset holders. This enables to assume away the debt and equities of firms. The income of asset holders net of investment pI is channeled in new deposits $\dot{D}$ (Eq 13).

Capitalists:

$$Y_c = rpK + \alpha_{\pi_b}\Pi_b + i_dD \quad (11)$$

$$S_c = Y_c \quad [C_c = 0] \quad (12)$$

$$\dot{D} = Y_c - pI \quad (13)$$

The equations describing the behaviour of firms are kept simple, as we focus here on worker debt. Firms produce a good Y according to fixed proportion technology. The quantity of labour and capital used by firms depends on the fixed proportion z and y^p . Labor demand L^d is a proportion z of output, with z being labour productivity⁹. Capital K is used in a proportion $Y/(uy^p)$, with u the rate of capacity utilization and Y^p the potential production of firms.

⁸ ξ_λ is a positive parameter ensuring that the mark up is positive.

⁹Labour productivity is a parameter and z is normalized to one through out the paper.

Entreprises:

$$L^d = Y/z \quad (14)$$

$$u = Y/Y^p \quad (15)$$

$$y^p = Y^p/K = cst \quad (16)$$

$$\dot{K} = I - \delta K \quad (17)$$

$$I/K = \alpha_r(r - r_0) + n \quad (18)$$

$$r = \frac{pY - wL^d}{pK} - \delta \quad (19)$$

The stock of capital increases with investment and decreases with capital depreciation at a rate δ . The rate of investment I/K depends positively on the deviation of profit rate r with respect to its long run value r_0 . At the steady state, investment equals n (the exogenous component of investment). Profit is an accounting identity equal to income from selling production Y minus labor costs and capital depreciation.

In line with Bhaduri and Marglin (1990), income distribution between labour and capital gives rise to wage-led and profit-led demand regimes. Although wages increase consumption, they also reduce profits and investment. The overall effect on aggregate demand is ambivalent depending on propensity of workers to consume of their labour income, and the propensity α_r of firms to invest out of their profits.

Wage price interaction:

$$\hat{w} = \beta_w(e - \bar{e}) + \kappa_w \hat{p} + (1 - \kappa_w)\bar{\pi}, \quad e = L^d/L = y/(zL) \quad (20)$$

$$\hat{p} = \beta_p(u - \bar{u}) + \kappa_p \hat{w} + (1 - \kappa_p)\bar{\pi}, \quad u = Y/Y^p = y/y^p \quad (21)$$

The demand typology is combined with a double Phillips curve for nominal wages w and prices p . The wage-price interaction follows Rose (1967) and is used in Chiarella et al. (2003) to model a Goodwinian conflict over income distribution (Goodwin, 1967). In this formulation, wages and prices are adjusted to some measure of labour market and goods market disequilibrium. Nominal wages w adjusts at a speed β_w to the discrepancy between the employment rate e and its long run value $\bar{e}$. The employment rate is the ratio between labour demand L^d and the active population L . The active population is normalized to one for simplicity: Accordingly, the dynamic of labour demand is used as a proxy for the change in employment. This assumption reduces the number of dynamic equations in contrast with Charpe et al. (2009), where the growth rate of the active population was

endogenous. Prices p adjusts at a speed β_p to the discrepancy between capacity utilization u and its long run value $\bar{u}$.

Nominal wages and prices also adjust to a weighted average of cost push elements. These costs push elements are made of model consistent cross over inflation $\kappa_w \hat{p}$ and wage $\kappa_p \hat{w}$, as well as a measure of the inflationary climate $\bar{\pi}$ (which is constant here). More complex formulations of the wage-price dynamic, including endogenous inflationary climate or an error correction term may be found in Chiarella et al. (2005).

Equilibrium condition:

$$Y = Y^d = C_w + I + \delta K \quad (22)$$

Lastly, macroeconomic closure is given by equality between aggregate supply and aggregate demand. Aggregate demand is defined as the sum of aggregate consumption and investment.

3 Reduced form equations and steady states

The model can be reduced to a three dimension dynamic system, made up of the equations for real wage, debt and deposits (equations 23 to 25)¹⁰. The reduced form of the model is found by de-trending all variables by the stock of capital. For instance, $\lambda = \frac{\Lambda}{pK}$ and $\dot{\lambda} = \left(\frac{\dot{\Lambda}}{pK}\right)$. The reduced form equation for the real wage is simply $\hat{\omega} = \hat{w} - \hat{p}$.

$$\dot{\omega} = \omega \kappa \left((1 - k_p) \beta_w \left(\frac{y}{z} - \bar{e} \right) + (k_w - 1) \beta_p (u - \bar{u}) \right) \quad (23)$$

$$\dot{\lambda} = (c - 1) y_w - \varphi_\lambda \lambda - \lambda (\hat{p} + g_k) \quad (24)$$

$$\dot{d} = r + i_d d + \alpha_{\pi_b} (i_\lambda \lambda - i_d d) - g_k - d (\hat{p} + g_k) \quad (25)$$

The following definitions must be inserted in the above three dynamic equations: household income y_w , investment g_k , growth rate of prices $\hat{p}$, capacity utilization u , labour demand l^d , profit rate r , interest on debt i_λ , interest on deposits i_d , the propensity to consume c ,¹¹ default rate φ_λ , banks' net wealth w_b^n and output y :

¹⁰Appendix 10.2 shows that there are two equations for deposits, one being redundant. It also suggests that there is one redundant equation out of the three dynamic equations for bank balance sheet: $\dot{\lambda}$, $\dot{D}$ and $\dot{W}_b^n$. We choose to work with the dynamic equations for debt and deposits and to define W_b^n as the difference between the two. This solution makes the proofs of the stability conditions easier to compute than in the alternative case, with debt and bank net wealth as the dynamic equations and deposits as the difference between the two.

¹¹We now assume that $c_0 = c_e + 1$ given that the dynamic equation for debt includes $(c - 1)y_w$.

$$\begin{aligned}
y_w &= \omega l^d - i_\lambda \lambda \\
g_k &= i_r(r - r_0) + n \\
\hat{p} &= \kappa \left[\beta_p(u - \bar{u}) + \kappa_p \beta_w(e - \bar{e}) \right] + \bar{\pi} \\
u &= y/y^p \\
l^d &= y/z \\
r &= y - \omega l^d - \delta \\
i_\lambda &= i_d + \xi_\lambda + \beta_{bis} \left[\left(\frac{w_{b0}^n}{\lambda_0} - \frac{w_b^n}{\lambda} \right) \right] \\
i_d &= \text{cst} \\
\kappa &= 1/(1 - \kappa_w \kappa_p) \\
c &= c_0 + \beta_c [w_b^n - w_{b0}^n] \\
c_0 &= c_e + 1 \\
\varphi_\lambda &= \beta_\varphi(y_{w0} - y_w) + n_\varphi \\
w_b^n &= \lambda - d \\
y &= \frac{n + \delta - i_r(\delta + r_0) - i_\lambda \lambda [c_0 + \beta_c [w_b^n - w_{b0}^n]]}{1 - i_r(1 - \omega/z) - \omega [c_0 + \beta_c [w_b^n - w_{b0}^n]]/z}
\end{aligned}$$

The IS curve is found by solving the equation for the equilibrium condition on the goods market for y and gives the equation for y ¹². The steady states are denoted with the subscript ₀ and are presented in Appendix 10.4.

4 Stable (wage-led) real sector and debt-deflation trends

We start the stability analysis a simple two dimension case, in which there is no debt default and bank performances do not affect credit supply $\beta_\varphi = \beta_{bis} = \beta_c = 0$. We obtain the following system of two equations made up of the dynamic of income distribution (the real wage) and debt:

$$\dot{\omega} = \omega \kappa \left((1 - k_p) \beta_w \left(\frac{y}{z} - \bar{e} \right) + (k_w - 1) \beta_p (u - \bar{u}) \right) \quad (26)$$

$$\dot{\lambda} = (c - 1) y_w - \lambda (\hat{p} + g_k) \quad (27)$$

¹²See Appendix 10.3 for the detailed computation

with the following IS curve: $y = \frac{n+\delta-i_r(\delta+r_0)-i_\lambda \lambda c_0}{1-i_r(1-\omega/z)-\omega c_0/z}$.

The different interactions between debt, output, income distribution and prices are illustrated in Fig 10.1. A higher debt to capital ratio has a negative impact on output as shown by the IS curve. Interest payments involve a redistribution of income from workers-borrowers with a high propensity to spend to capitalists-lenders with a low propensity to spend. This effect is similar to Dutt (2006). The recessionary effect of higher debt stabilizes the accumulation of debt because it reduces the disposable income of workers as well as their level of consumption.

The debt dynamic also interacts with functional income distribution. The primary distribution of income between labour and capital (captured by the variable ω) has contrasting effects on the IS curve, negatively affecting investment and positively affecting consumption. The wage-led, profit-led typology depends on the relative propensity to spend out of profit i_r and out of wages c_0 .

The relative speed of adjustment of nominal wages and prices $\beta_w, \beta_p, \kappa_w, \kappa_p$ gives rise to two types of real wage dynamics. On the one hand, the real wage is said to be goods market-led when prices are more flexible than nominal wages. On the other hand, the real wage is said to be labour market-led when prices are less flexible than nominal wages.

The interaction between the two typologies of aggregate demand and the two typologies of real wage give rise to four institutional configurations. Only two of these configurations are stable. Wage-led aggregate demand is stable if real wages are goods market-led. Profit-led aggregate demand is stable if real wages are labour market-led (see Chiarella et al. (2005) for a detailed presentation). The institutions shaping the properties of the real sector are crucial to the overall stability of the economy, as they impact the ability of the real sector to absorb financial shocks.

A detailed discussion of the different cases and their stability properties can be found in Charpe et al. (2009) and Charpe et al. (2011). This paper is limited to the case in which the IS curve is demand-led $c_0 > i_r$ and in which the sign of the numerator and denominator of the IS curve are positives. The propensity to consume c_0 is larger than one to ensure a positive debt to capital ratio at the steady state. The numerator and denominator of the IS curve can, therefore become negative if i_r is large, and generate a variety of complex situations.

A last aspect of the model is the possibility of having a debt-deflation spiral. There are two ambivalent effects related to price flexibility and whose relative strength determines the overall stability of the system of equation. As shown above, price flexibility stabilizes the real sector when aggregate demand is wage-led. Conversely, price flexibility has defla-

tionary effects on household debt. Price deflation increases the value of real debt, which affects negatively household income and consumption.

Proposition 1 (Stability of income distribution and debt)

Assume that the parameters are chosen such that the following condition holds:

$$1 + (1 - i_r)(1 - \omega_0)/\omega_0 > c > 1 > i_r.$$

Assume in addition that the parameters for the wage price spiral $\beta_p, \beta_w, k_p, k_w$ are such that the following condition holds:

$$\left[(1 - k_p)\beta_w + (k_w - 1)\frac{\beta_p}{y^p} \right] < 0$$

Lastly assume that the parameter i_r is such that the following condition holds:

$$1/(1 + \lambda_0) > i_r$$

The system of equation 26 – 27 is locally stable. These assumptions ensure a negative trace and a positive determinant¹³ (See appendix 10.5 for the detailed computation).

The first condition $1 + (1 - i_r)(1 - \omega_0)/\omega_0 > c > 1 > i_r$ implies that there is a positive debt to capital $c > 1$, that aggregate demand is wage-led $c > i_r$ and that the numerator and denominator of the IS curve are positive. The second condition $\left[(1 - k_p)\beta_w + (k_w - 1)\frac{\beta_p}{y^p} \right] < 0$ involves goods market-led real wages: prices are relatively more flexible than nominal wages. These first two conditions generate a stable dynamic between the demand regime and functional income distribution. This is reflected by the negative sign of the entry J_{11} . The ceiling on the propensity to invest from profits $i_r < 1/(1 + \lambda_0)$ ensures that the debt to capital ratio (in the absence of price effect) does not increase because of low capital accumulation following a positive shock on debt.

The trace ($J_{11} + J_{22}$) is negative despite the positive sign of the entry $J_{22} > 0$. Debt has a ponzi trend and is self-increasing. The negative impact of debt on output slows price inflation and/or generates price deflation. Lower price inflation or price deflation increases the real debt to capital ratio. The trace is negative if the debt deflation effect associated with price flexibility, which enters J_{22} is smaller than the stabilizing effect of price flexibility on the real sector, which enters J_{11} . Prices have opposite effects. On the one hand, they stabilize the wage-price dynamic. On the other hand, they produce debt-deflation spirals. The former effect is always greater than the latter in this model.

¹³A system of two dynamic equations is stable if its trace ($J_{11} + J_{22}$) is negative and its determinant is positive. The entry J_{ij} is the element of the Jacobian matrix made of the first derivative of equation i with respect to variable j .

The determinant is positive $|J| > 0$, ensuring the stability of the economy. The entry J_{12} is positive. An increase in debt increases real wages. The contractionary effect of debt tends to push down nominal wages. However, given that prices are relatively more flexible than wages, the real wage increases. Lastly, the entry J_{21} is negative. Higher real wages reduce indebtedness. On the one hand, higher wages feed consumption and the need for funds by households. On the other hand, in a wage-led economy higher wages produce inflation which stabilizes real debt. A change in functional income distribution away from labour generates an increase in debt. This increase is, however, rather different than the mechanism put forward by the demand side views, as the increase in higher real debt results from a price effect. In addition, there is no debt-financed consumption boom as real debt is recessionary.

We briefly describe the calibration of the main parameters used for the numerical simulations. The propensity to consume is set at $c = 1.1$ to ensure a positive debt to capital ratio. The rate of capacity utilization $\bar{u}$ is 0.9 in the long term, while investment is growing at $n = 3.5\%$ at the steady state. Capital depreciates at a rate of $\delta = 5\%$, while the productivity parameter is set to $z = 1$ and the interest rate is 2%. The speed of adjustment of nominal wages is relatively slower than that of prices $\beta_w = 0.15$, $\beta_p = 0.2$. The cost push elements are translated into wages and prices at the same speed $\kappa_w = \kappa_p = 0.5$.

Numerical simulations are consistent with the results of the stability conditions (figure 10.2). The ponzi trend and the stabilizing real wage appears clearly in subfigure 10.2(a) and 10.2(b). The positive shock on debt places the debt dynamic on an increasing trend due to the deflationary prices (corresponding to the hump shaped part of the impulse response for debt). However, the increase in real wages sustains aggregate demand and counteracts the deflationary spiral. The economy converges back to its steady state. Despite the initial cumulative effect of debt on itself, the real sector absorbs the positive shock on debt. The recessionary effect of debt on output and the implied deflation of prices is not explosive as the real wage increases and stabilizes aggregate demand. Nominal wage flexibility is destabilizing, as shown by the maximum real part of eigenvalues¹⁴, which turns positive for large values of β_w and κ_w (subfigure 10.2(c) and 10.2(e)). Conversely, price flexibility is stabilizing as the eigenvalues are negative for large values of β_p and κ_p (subfigure 10.2(d) and 10.2(f)).

¹⁴A system is stable if the eigenvalues are negative. Drawing the maximum real part of eigenvalues with respect to different values of a parameter is a convenient way to show for which critical value of a parameter the economy becomes unstable.

5 Credit rationing and consumption boom

The goal of this section is to understand the impact of the performance of banks on credit rationing, debt accumulation and consumption. In the previous case, banks were passive, accommodating the demand for loans from households. Conversely, here, banks are proactive and modify their loan policy depending on their performance $\beta_c \neq 0$. In a first step, we assume that banks only adjust the quantity of credit supplied in response to changes in their net wealth, keeping the mark-up on interest rate constant $\beta_{bis} = 0$. For the sake of simplicity, we also consider the case in which households do not default on debt $\beta_\varphi = 0$. Lastly, setting β_w and β_p equal to zero, we obtain a two dimensional system of equations consisting of household debt and bank deposits (Eq 28 and Eq 29). The wage share is fixed at its steady state. Income distribution between labour and capital does not feedback with debt accumulation.

$$\dot{\lambda} = (c - 1)y_w - \lambda(\bar{p} + g_k) \quad (28)$$

$$\dot{d} = r + i_d d + \alpha_{\pi_b}(i_\lambda \lambda - i_d d) - g_k - d(\bar{p} + g_k) \quad (29)$$

The main feedback channels are displayed in figure 10.3. In the absence of credit rationing and price deflation, debt tends to be self-stabilizing. Increases in debt raise interest payments, which reduce household income and consumption decisions. Taking into account credit rationing affects the dynamic of debt accumulation. Increase in debt also improve bank profits and net wealth. Banks therefore relax credit rationing, which feeds household consumption. Bank proactive behaviour generates a destabilizing feedback channel in which debt and consumption feed each other, leading to over-indebtedness. Debt feeds the consumption boom, an effect similar to Dutt (2006). On the other hand, the feedback loop associated with deposits is self-stabilizing. Any increase in deposits reduces bank net wealth. The tightening of credit in return reduces aggregate demand and deposits of asset holders.

Proposition 2 (Stability of credit rationing)

Assume that the parameters are chosen such that the following condition holds:

$$1 + (1 - i_r)(1 - \omega_0)/\omega_0 > c > 1 > 1/(1 + \lambda_0) > i_r.$$

Assume in addition that the following condition holds: $\beta_c < \frac{c_0 i_\lambda}{y_w}$

The system of equations 28 and 29 is locally stable. These assumptions ensure a negative trace and a positive determinant (See Appendix 10.6 for the detailed calculation).

The first condition $1 + (1 - i_r)(1 - \omega_0)/\omega_0 > c > 1 > 1/(1 + \lambda_0) > i_r$ is similar to Proposition 1 and ensures wage-led aggregate demand, positive debt to capital ratio and a positive numerator and denominator of the IS curve. It also sets a ceiling on the propensity to invest from profits, which shapes the dynamic of the debt to capital ratio over the cycle. The value of β_c has ambivalent effects on the trace. On the one hand, it enters negatively the entry J_{22} , as higher deposits limit banks' credit supply. On the other hand, it enters positively the entry J_{11} , as credit supply feeds the economic boom and bank performance. The net effect depends on the steady state value of bank net wealth, which is positive by assumption, ensuring a negative trace. The sign of the determinant depends, however, on β_c and is positive only if $\beta_c < \frac{c_0 i_\lambda}{y^w}$ holds. The entry J_{12} is negative, through the impact of deposits on credit supply and debt accumulation. Higher debt, however, tends to boost deposits and turn the sign of the entry J_{21} positive if the credit channel $\beta_c y^w$ is stronger than the recessionary effect of higher interest payments $c_0 i_\lambda$.

Figure 10.4 shows the dynamic of the economy when credit supply depends on bank performance. Parameters are similar to that of the previous simulation. A first difference is that the coefficient driving credit rationing is now positive and different from zero $\beta_c = 0.05$. A second difference is that income distribution does not feed back with debt accumulation ($\beta_w = \beta_p = \kappa_p = \kappa_w = 0$). The main result is that debt is strongly procyclical as shown on the left panel. Previously, higher debt would impact negatively on household income and consumption, reducing output. In the present case, following a positive shock on debt, higher debt feeds a consumption boom, as bank performance improves and loosens credit rationing. Bank performance appears to be pro-cyclical as shown on the center panel of figure 10.4. Lastly, the positive feedback loops between debt and consumption is potentially destabilizing. For values of β_c larger than 0.4, the sensitivity of credit supply to bank performance produce explosives forces (positive eigenvalues) leading to over-indebtedness.

6 Debt default

6.1 Income distribution and debt default

In this section, we consider the case of household default due to over-indebtedness. The case considered in Section 4 showed that over-indebtedness may trigger a debt deflation spiral. A natural extension is to ask whether debt default might be a way to overcome the debt crisis. The impact of debt default on household budget constraints is studied here in isolation from the negative effects that default can have on bank profitability (see Section

6.2). In contrast with Section 4, the case of debt default implies $\beta_\varphi \neq 0$. The system of equations is still two dimensional (eq 30 and 31)¹⁵.

$$\dot{\omega} = \omega \kappa \left((1 - k_p) \beta_w \left(\frac{y}{z} - \bar{e} \right) + (k_w - 1) \beta_p (u - \bar{u}) \right) \quad (30)$$

$$\dot{\lambda} = (c - 1) y_w - \varphi_\lambda \lambda - \lambda (\hat{p} + g_k) \quad (31)$$

Figure 10.5 shows the main feedback channels associated with debt default. Debt default has a stabilizing effect on the debt dynamic. Debt default reinforces the stabilizing loops between debt accumulation, household income and consumption decisions. Higher debt increases the default rate through the negative impact of interest payments on household income. In return, the level of indebtedness drops with the rate of debt default. Debt default limits the transfer of income associated with interest payments from workers-borrowers with a high propensity to spend to capitalists-lenders with a low propensity to spend. It is in fact a transfer of loses from borrowers to lenders. However, debt default has an indirect perverse effect. Goods market-led real wages increase with indebtedness, as higher wages improve household income and slow down debt default.

Proposition 3 (Stability of income distribution and debt default)

In addition to the assumption made in proposition 1, assume lastly that the following condition holds: $\beta_\varphi < \frac{1-i_r(1+\lambda_0)}{\lambda_0 i_r}$

The system of equation 30 – 31 is locally stable. These assumptions ensure a negative trace and a positive determinant (See Appendix 10.7 for the detailed computation).

The speed at which households default on debt β_φ enters the trace negatively through J_{22} . However, β_φ enters J_{21} positively and tends to shift its sign from negative to positive. To ensure that the indirect destabilizing effect of debt default through the adjustment of wages is small, β_φ must be lower than $\frac{1-i_r(1+\lambda_0)}{\lambda_0 i_r}$.

Figure 10.6 shows the numerical simulations for the case in which households default on debt. Parameters are identical to that of the simulation in Section 4, with the exception of β_φ , which is now equal to 0.2. Following a positive debt shock, debt default significantly reduces the ponzi trend in the debt dynamic (subfigure 10.6(a)). The period over which debt is on a self-increasing trajectory is much shorter. The discrepancy is obvious when the debt dynamic is compared to that corresponding to the previous numerical simulation $\beta_\varphi =$

¹⁵with $\varphi_\lambda = \beta_\varphi (y_{w0} - y_w) + n_\varphi$

0. Default also reduces debt deflation significantly (subfigure 10.6(b)). The ambivalent effect of debt default is visible for large values of β_φ . The maximum real part of eigenvalues turns positive for values of β_φ larger than 8 (subfigure 10.6(c)).

6.2 Credit rationing and debt default

This subsection addresses the effect of debt default on debt accumulation in the event of credit rationing, taking into account that debt default now has an impact on bank performance. The main difference from section 5 is that β_φ is no longer equal to zero and now feeds back into the equation for debt accumulation. This section also differs from section 6.1 in which debt default contributed to stabilize debt accumulation. Here, on the contrary, debt default is likely to be destabilizing given that debt default is a loss for banks and generates a tightening of credit supply. The system of equations is still two dimensional (eq 32 and eq 33)¹⁶.

$$\dot{\lambda} = (c - 1)y_w - \varphi_\lambda \lambda - \lambda(\bar{p} + g_k) \quad (32)$$

$$\dot{d} = r + i_d d + \alpha_{\pi_b}(i_\lambda \lambda - i_d d) - g_k - d(\bar{p} + g_k) \quad (33)$$

The main feedback channels associated with debt default and bank behavior are illustrated in figure 10.7. The feedback channel involved in section 6.1, in which default reduces interest payments and debt accumulation, is still present. The interaction between debt default and credit rationing is however destabilizing. Debt default reduces bank net equity and produces a tightening of credit. The negative impact on aggregate demand further reduces household income and leads to additional default. This feedback channel captures the vicious circle at work during financial crises, in which the tightening of credit, the deterioration of economic activities and the accumulation of non-performing loans in bank balance sheets is self-increasing. In such a case, the interventions of monetary and fiscal authorities are necessary to avoid the collapse of the financial system, through lender of last resort activities or through a mechanism to collect and re-cycle loan losses and clean up bank balance sheets.

Lastly, the feedback channel associated with bank deposits is stabilizing, similar to the result of section 5, and does not directly interact with debt default. The banking crisis considered in this section is not of the type of a bank run. The deterioration of bank balance sheet is not resulting from the depletion of deposits. The banking crisis occurs due to the impact of debt default on the asset side of the balance sheet.

¹⁶with $\varphi_\lambda = \beta_\varphi(y_{w0} - y_w) + n_\varphi$

Proposition 4 (Stability of debt default and credit rationing)

*In addition to the assumptions made in proposition 2, assume further that:
 β_φ and α_{π_b} are small and that $n_\varphi = \xi_\lambda$,
then, the system of equations 32 and 33 is locally stable. These assumptions ensure a negative trace and a positive determinant (See Appendix 10.8 for the detailed calculation).*

The sensitivity of credit supply to the performances of banks β_c has similar effects on the trace and the determinant as in section 5. β_c enters the trace negatively but needs to be smaller than $i_\lambda c_0/y_{w0}$ to ensure a positive determinant. The speed at which households default on debt β_φ enters both the entries J_{11} and J_{12} , while the entries J_{21} and J_{22} are similar to the corresponding entries of section 5.

The impact of debt default on the sign of J_{11} reflects two opposite mechanisms: $-\beta_\varphi \lambda_0 [i_\lambda [1 - i_r(1 - \omega_0)] - \omega_0 y_{w0} \beta_c]$. The first element in the bracket has a negative impact on J_{11} and accounts for the feedback channel discussed in section 6.1. The second element in the bracket has a positive effect on J_{11} and takes into account the negative impact of default on credit supply. The strength of this last effect depends on the value of β_c . However, the entry J_{11} is negative for small values of β_φ .

The entry J_{12} is now augmented by the element $-\beta_c \omega_0 y_{w0} \lambda_0 \beta_\varphi$, which captures the effect of a change in bank liability on debt accumulation in the presence of credit rationing and debt default. This new element is negative and does not change the sign of the entry: $J_{12} < 0$. Given that all the entries are negative¹⁷, the sign of the determinant is unclear. Simplifying the determinant enables us to show that the determinant is positive for small values of β_φ ¹⁸. This result points to the destabilizing effect of credit rationing in the presence of debt default. To simplify the determinant, two assumptions were made. First, it is assumed that the share of bank profit distributed to capitalists is limited (α_{π_b} is small). Second, the long term rate of debt default n_φ is equal to the long term mark-up of banks ξ_λ . These assumptions enable us to simplify the entries J_{12} and J_{22} and to find an analytically tractable expression for the sign of the determinant.

Figure 10.8 illustrates the influence on the volatility of output of debt default in the event of credit rationing. The main parameters are similar to that of section 5, except β_c , which is equal to 0.1, β_φ , which is equal to 0.8 and α_{π_b} , which is equal to 0.15. Figure 10.8(a) shows that the rate of debt default is counter-cyclical and is likely to

¹⁷We know from section 5 that the entries J_{21} and J_{22} are both negative.

¹⁸See section 10.8

produce volatility to the extent that it affects the credit supply. Figure 10.8(b) highlights the impact of an increase in credit rationing and debt default on the volatility of output. The baseline case is obtained by making use of the set of parameters described above (blue line). Increasing the speed of adjustment of debt default β_φ to 1.2 increases the volatility of output, due to the negative effects of debt default on credit supply. Similarly, increasing the sensitivity of credit supply to debt default by increasing β_c to 0.1025 generates wider business cycle fluctuations. The fragility of a financial system in the presence of large loan losses is confirmed by the maximum real part of eigenvalues for β_c and β_φ , which both turn positive very rapidly.

7 Prudential ratio and the endogenous mark-up

This section discusses the ability of prudential regulation to reduce financial instability. The system of equations considered consists of equations 34 and 35 but, β_{bis} is no longer equal to zero. Banks not only adjust the quantity of credit granted to households, but also adjust the interest rate mark-up according to the BIS capital adequacy ratio (CAR). The CAR is a prudential ratio measuring the risk of exposure of banks. It consists of the ratio of a bank's capital to its risk, which is given here by the ratio between banks' net wealth (or bank equity) and its risky assets (the stock of loans to households): $CAR = \frac{w_b^n}{\lambda}$. Banks raise the interest rates on debt when the CAR drops below its steady state value. We consider two cases, where income distribution does not feedback with debt accumulation. In the first case, there is no credit rationing. New debt closes the gap between household consumption and income. Bank performances affect the dynamic of debt to the extent that the capital adequacy ratio impacts on interest payments from workers. In the second case, bank performances affect debt accumulation in two ways through the endogenous mark up and through credit rationing.

$$\dot{\lambda} = (c - 1)y_w - \lambda(\bar{p} + g_k) \quad (34)$$

$$\dot{d} = r + i_d d + \alpha_{\pi_b}(i_\lambda \lambda - i_d d) - g_k - d(\bar{p} + g_k) \quad (35)$$

$$i_\lambda = i_d + \xi_\lambda + \beta_{bis} \left[\left(\frac{w_{b0}^n}{\lambda_0} - \frac{w_b^n}{\lambda} \right) \right]$$

Figure 10.9 illustrates the main feedback channels associated with the capital adequacy ratio of banks and the endogenous mark-up on household debt. An increase in debt generates two unstable mechanisms. First, higher debt increases the capital adequacy ratio leading to a drop in bank mark-ups. Lower interest rate payments increase household

income and consumption, which leads to further increases in debt. Second, a lower mark-up impacts negatively on bank profits and dividends. Lower deposits further improve banks' capital adequacy ratio and further reduces the mark-up. The CAR has here a pro-cyclical impact on the economy, which amplifies business cycle oscillations. This result is, however, likely to be reversed when credit rationing is involved. The reduction in the interest rate mark up following an improvement of the CAR is detrimental to bank performance. In response, banks squeeze credit, which stabilizes the accumulation of debt.

Proposition 5 (Stability of prudential ratio ($\beta_c = 0$))

Assume that the parameters are chosen such that the following condition holds:

$$1 + (1 - i_r)(1 - \omega_0)/\omega_0 > c > 1 > 1/(1 + \lambda_0) > i_r.$$

Assume furthermore that the following condition holds: $\beta_{bis} < \frac{i_\lambda}{d_0}$

assume that: $i_r < \frac{c-1}{(1-\omega_0)[c(1+\lambda_0)-1]}$

and that: α_{π_b} is small

The system of equations 34 and 35 is locally stable. These assumptions ensure a negative trace and a positive determinant (See Appendix 10.9 for the detailed calculation).

The first assumption ensures that aggregate demand is wage-led and that the numerator and denominator of the IS curve are both positive. The second and third assumptions generate a negative entry J_{11} . In particular, an increase in debt does not trigger an unstable dynamic of debt accumulation driven by the impact of a lower interest rate on household net income. Hence the upper ceiling on β_{bis} . The sign of J_{22} is negative too, given that we already know from Proposition 4 that the share of profits α_{π_b} distributed to capitalists is small. The trace is thus negative.

The entry J_{21} is also negative given that we have already assumed that β_{bis} and α_{π_b} are small. The recessionary impact of interest payments is not counterbalanced by a lower interest rate. In addition, bank profits are not distributed on a large scale and are not transformed into higher deposits. Lastly, we also know that J_{12} is negative, which leaves the sign of the determinant unclear. However, implementing the simplifications described in Appendix 10.9 ensures a positive determinant. The main result is that the capital adequacy ratio has, in the absence of credit rationing, an accelerating effect on debt accumulation through its positive impact on household net income.

Figure 10.10 displays the dynamic of the BIS ratio and the interest rate following a positive shock on debt. The parameters are similar to the parameters of the previous simulation with the exception of β_{bis} , which is now equal either to 0.035 or 0.435. Another

difference is that the effect of the BIS ratio is studied in isolation with credit rationing $\beta_c = 0$. The main result is that in line with the stability condition and with Figure 10.9, an increase in the sensitivity of the mark-up to the BIS ratio reduces the stability of the system. Higher debt improves the BIS ratio and reduces the interest rate, which feeds household demand for new loans. Increasing the sensitivity of β_{bis} from 0.035 to 0.435 increases the time needed for the economy to converge back to the steady state. Similarly, eigenvalues turn positive for small values of β_{bis} .

Figure 10.11 shows the dynamic of the economy when both the mark up is endogenous and banks ration credits. The parameters are identical to that of the previous simulations except that $\beta_c = 0.105$. In such a case, a higher sensitivity of the mark up to the BIS ratio tends to stabilize the business cycle. In fact, given that the interest rate drops following a positive shock on debt, banks' profits also drop, which has a direct effect on banks' performances and banks' supply of credit. It appears clearly from the left panel as well as from the center panel that convergence is faster when β_{bis} is equal to 0.435 rather than 0.035. Similarly, the eigenvalues are now increasingly negative for large values of β_{bis} . This results stand in contrast with the previous finding that the BIS ratio has pro-cyclical effect.

8 Three dimensional case

The demand side and the supply side explanations of household debt are not as opposed as it may appear. In real the world, both income effect and credit supply effect are likely to account for the rise in debt. This section considers the case where both income distribution and credit rationing are interacting. We study the properties of the three dimensional system of equations (eq 23 to eq 25) through numerical simulations.

In Figure 10.12, we compare two experiments. A baseline scenario illustrates the case of credit rationing without income distribution. The wage share is constant and the parameters are similar to the parameters of section 5 (except that α_{π_b} is now equal to 0.5). In the baseline scenario, credit rationing produces a debt financed consumption boom, which is highly unstable. Introducing income distribution ($\beta_w = 0.15$, $\beta_p = 0.2$, $\kappa_w = \kappa_p = 0.5$) produces complex dynamics. The economy is converging at a slow path and business cycle oscillations are taking place around this trend.

9 Conclusion

This paper presents a model to better understand the drivers of financial instability related to household debt. In particular, this model focuses on two competing views of household debt. The demand side explanation of household debt points to the deterioration of household income in explaining the increase in debt. The supply side explanation points to the key role of credit rationing to account for the rise in household debt. Although these two views may seem opposed, they are in fact closely inter-related. The demand side view assumes that financial liberalization is a necessary condition prior to the increase in household debt. The supply side view on the contrary, assumes that labour income falls short of household spending.

There are four main results. First, the standard Keynesian consumption function does not account for the debt financed consumption boom stressed by the demand side view. The recessionary effect of the income transfer between workers-borrowers with a high propensity to consume and capitalists-lenders with a low propensity to spend stabilizes debt accumulation, with the exception of a Fisher effect generated by changes in prices. The Keynesian consumption function could be extended to include autonomous spending or Veblen type consumption.

Second, credit rationing is more successful in accounting for the debt financed consumption boom. Credit supply is pro-cyclical and stimulates both consumption and indebtedness. Debt is strongly pro-cyclical despite its recessionary effect through interest payments. The main novelty here is to express credit supply as a function of bank performance, rather than borrower performance as in Dutt (2006). The model takes into account the boomerang effect of over-indebtedness on the stability of the financial system.

Third, the boomerang effect of over-indebtedness on financial institutions contributes to the rise of financial fragility. This mechanism is only at work in the event of credit rationing, as bank performances do not otherwise feedback onto the real economy. In particular, debt default stabilizes the economy in the case of the demand side view. Debt default reduces the transfers of income between borrowers and creditors generated by interest payments. Put differently, giving a "hair cut" to creditors is a viable and a fair way out of the crisis to the extent that it does not endanger the stability of the financial system. It stops the debt deflation spiral and places a share of the burden on creditors who benefited from large income transfers before the crisis.

Fourth, prudential regulation may contribute to financial stability under certain circumstances. The direct effect of CAR on the interest rate is similar to that of a financial

accelerator. This pro-cyclical effect is reversed in the case of credit rationing, as CAR negatively affects bank performance. Credits are less pro-cyclical and a brutal credit crunch is avoided. Capital adequacy ratio weakens the financial accelerator effect to the extent that it also has an impact on the quantity of credit supplied to households.

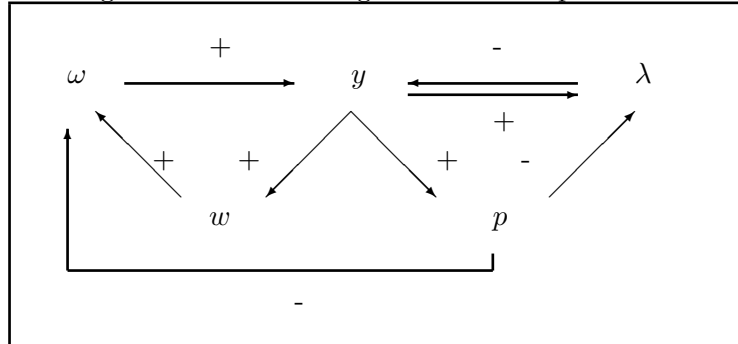
10 Appendix

10.1 Tables and figures

Table 1: Bank balance sheet

Balance sheet		Profits net of dividends and loan losses	
Λ	D	$i_\lambda \Lambda$	$i_d D$
	W_b^n		$\alpha_{\pi_b} \Pi_b$
			$\varphi_\lambda \Lambda$

Figure 10.1: Stabilizing real sector vs ponzi trend



With ω real wages, y output, w nominal wages, p prices and λ real debt to capital ratio. The arrow points to the causality between two variables and the sign (+ or -) indicates the nature of the causality. A feedback loop in which the product of the signs is positive (negative) indicates that the interaction between these variables is destabilizing (stabilizing).

Figure 10.2: Stable real sector and debt deflation trends

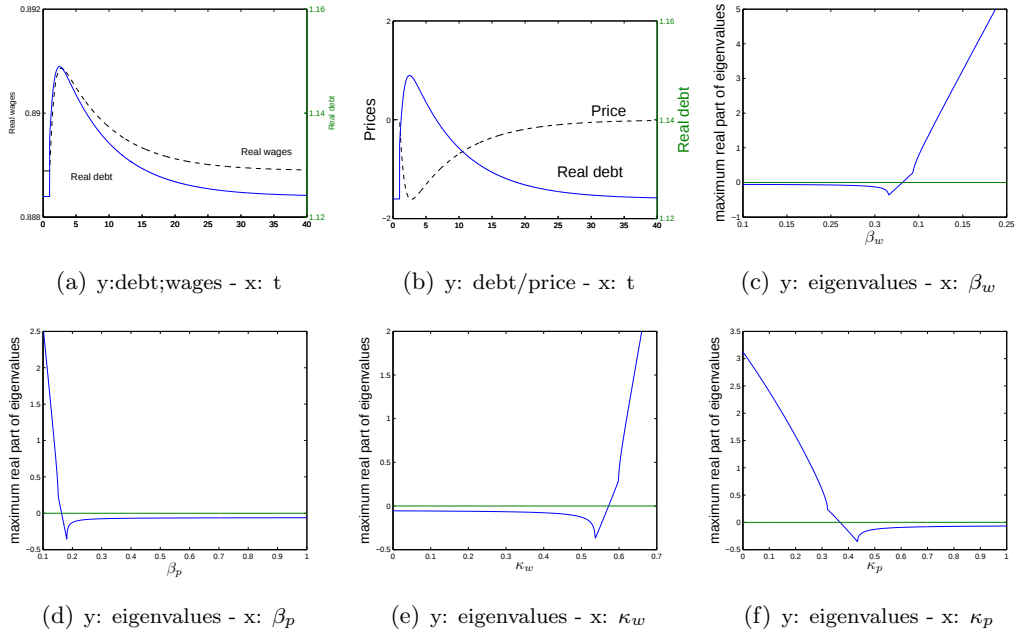
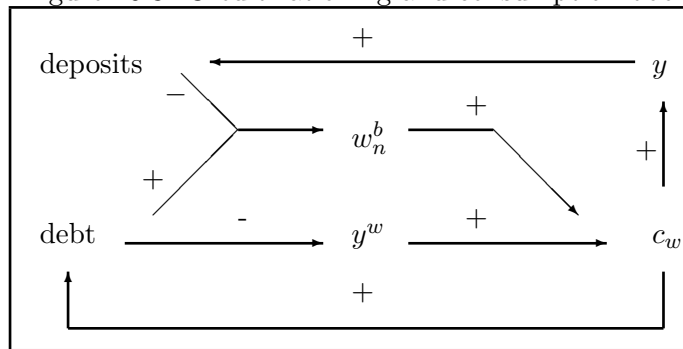
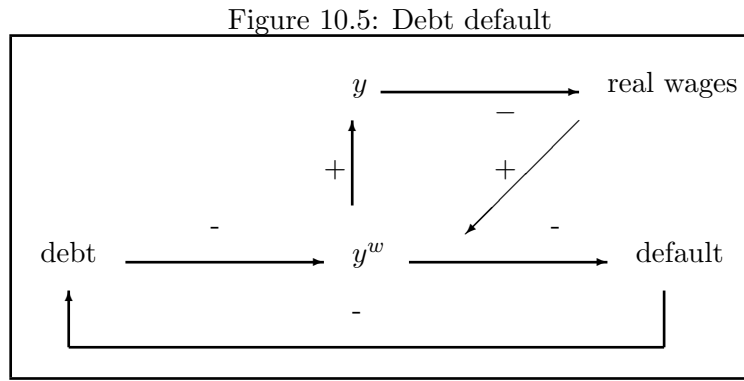
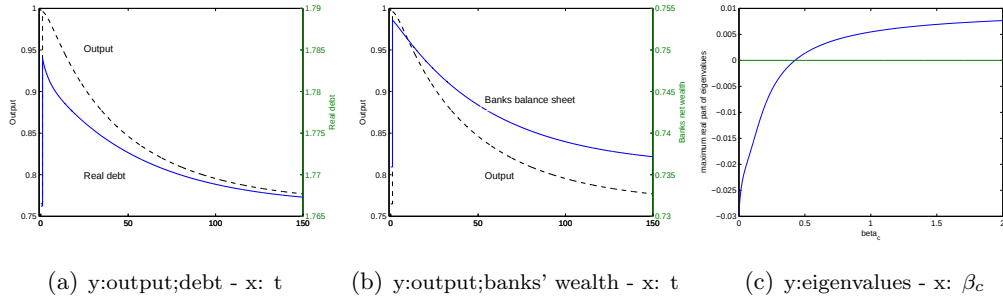


Figure 10.3: Credit rationing and consumption boom



With y^w the net income of workers, c_w consumption of workers, y output and w_n^b the net wealth of banks.

Figure 10.4: Bank performances, credit rationing and consumption boom



With y^w the net income of workers and y output.

Figure 10.6: Debt default

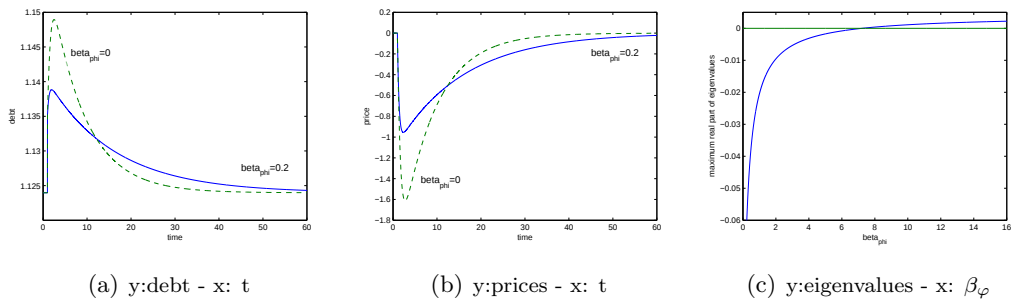
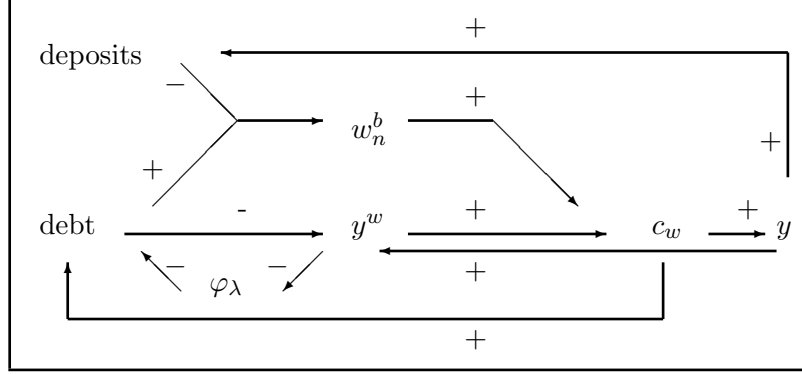


Figure 10.7: Debt default and the credit channel



With y^w the net income of workers, c_w consumption of workers, y output, w_n^b the net wealth of banks and φ_λ the rate of debt default.

Figure 10.8: Debt default and credit rationing

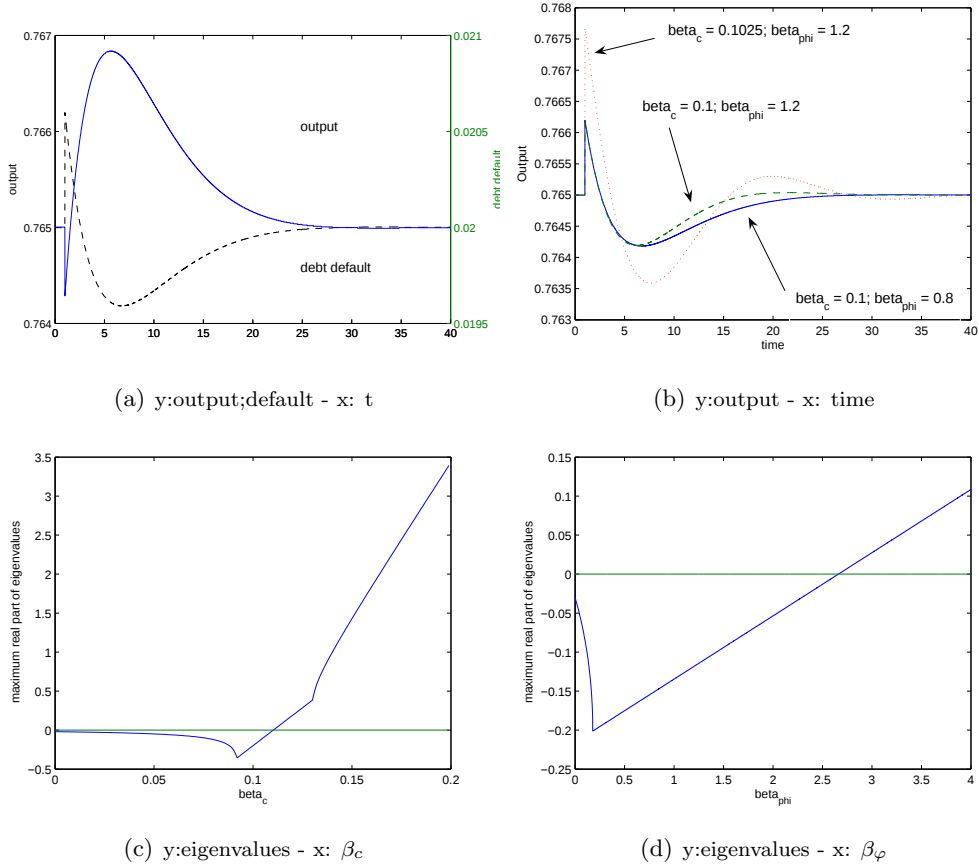
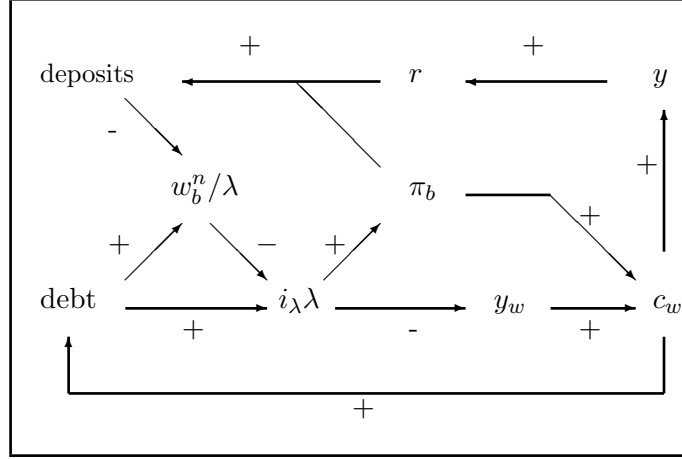


Figure 10.9: Interest rate channel



With $i_\lambda \lambda$ interest payments, y_w the income of workers, c_w the consumption of workers, w_b^n/λ the capital adequacy ratio, π_b the profit of banks, y output, r the profit of firms.

Figure 10.10: Pro-cyclical prudential ratio

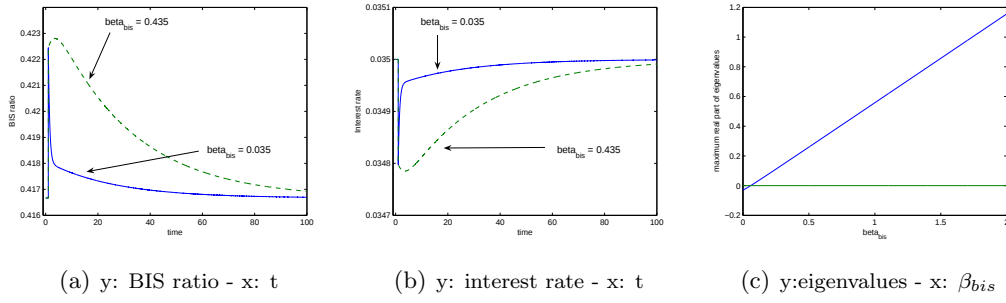


Figure 10.11: Credit rationing and counter-cyclical prudential ratio

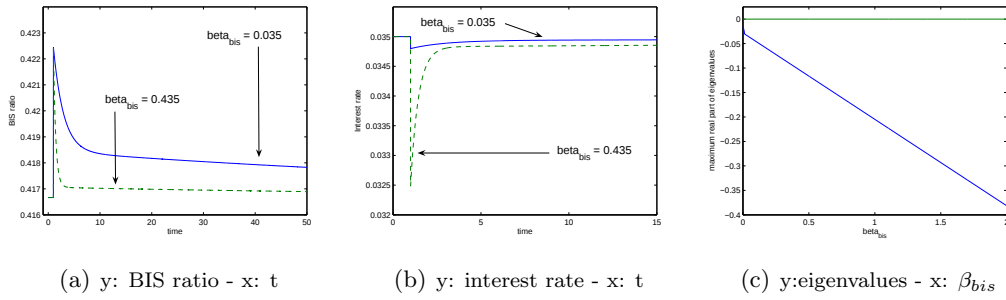
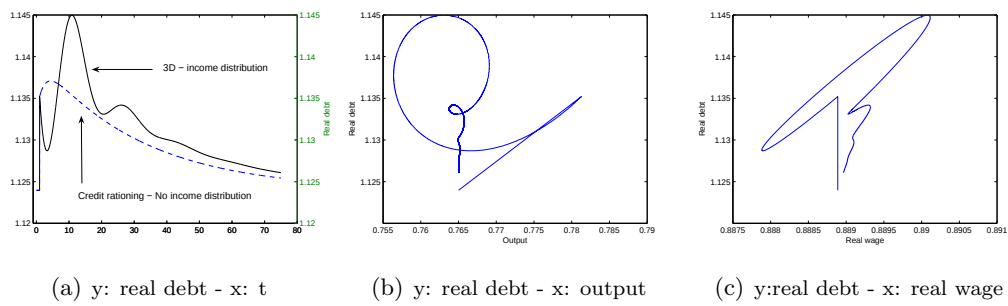


Figure 10.12: Credit rationing and income distribution



10.2 Endogenous money

It is possible to show that the two equations for money $\dot{D}$ are identical, which confirms the endogenous money hypothesis:

$$\begin{aligned}
\dot{\Lambda} - \dot{W}_b^n &= Y_c - pI \\
\dot{\Lambda} - (1 - \alpha_{\pi_b})\Pi_b + \varphi_\lambda \Lambda &= Y_c - pI \\
(c - 1)Y_w - (1 - \alpha_{\pi_b})\Pi_b &= rpK + \alpha_{\pi_b}\Pi_b + i_d D - pI \\
(c - 1)Y_w - i_\lambda \Lambda &= rpK - pI \\
(c - 1)Y_w - i_\lambda \Lambda &= pY - wL^d - \delta pK - pI \\
cY_w &= pY - \delta pK - pI \\
pC_w + \delta pK + pI &= pY
\end{aligned}$$

which always holds given equilibrium in the goods market.

10.3 The IS curve

The IS curve solves the goods market equilibrium.

$$\begin{aligned}
y &= c_w + g_k + \delta \\
&= \left[c_0 + \beta_c [W_b^n - W_{b0}^n] \right] y^w + i_r(r - r_o) + n + \delta \\
y &= \frac{n + \delta - i_r(\delta + \bar{r}) - i_\lambda \lambda \left[c_0 + \beta_c [W_b^n - W_{b0}^n] \right]}{1 - i_r(1 - \omega/z) - \omega \left[c_0 + \beta_c [W_b^n - W_{b0}^n] \right] / z} = \frac{N}{D}
\end{aligned}$$

10.4 Steady states

Steady states are as follow:

$$\begin{aligned}
y_0 &= y^p \bar{u}, \quad \bar{e} = y^p \bar{u}/z \\
y_{w0} &= \frac{y_0 - \delta - n}{c_0}, \quad \lambda_0 = \frac{(c-1)y_{w0}}{n + n_\varphi + \bar{\pi}} \\
\omega_0 &= z \frac{y_{w0} + i_\lambda \lambda_0}{y_0} = z \frac{y_{w0}}{y_0} \left(1 + i_\lambda \frac{c-1}{n + n_\varphi + \bar{\pi}}\right) = \frac{y_0 - n - \delta}{y_0} \\
r_0 &= (1 - \omega_0) \frac{y_0}{z} - \delta = n \\
\hat{p}_0 &= \bar{\pi}, \quad g_{k0} = n \\
d_0 &= \frac{r_0 + \alpha_{\pi_b} i_\lambda \lambda - n - \bar{\pi}}{n + \bar{\pi} - i_d(1 - \alpha_{\pi_b})} \\
w_{b0}^n &= \lambda_0 - d_0, \quad i_\lambda = i_d + \xi_\lambda
\end{aligned}$$

10.5 Stability proof: income distribution and debt

Assuming $\beta_\varphi = \beta_{bis} = \beta_c = 0$, we obtain a two dimensional system made of equations 26 – 27:

$$\dot{\omega} = \omega \kappa \left((1 - k_p) \beta_w \left(\frac{y}{z} - \bar{e} \right) + (k_w - 1) \beta_p (u - \bar{u}) \right) \quad (36)$$

$$\dot{\lambda} = (c - 1) y_w - \lambda (\hat{p} + g_k) \quad (37)$$

with the following IS curve and interest rate equations and propensity to consume:

$$\begin{aligned}
i_\lambda &= i_d + \xi_\lambda, \quad c = c_0 \\
y &= \frac{n + \delta - i_r(\delta + \bar{r}) - i_\lambda \lambda c_0}{1 - i_r(1 - \omega/z) - \omega c_0/z}
\end{aligned}$$

The Jacobian matrix has the following entries:

$$\begin{aligned}
J_{11} &= \omega_0 \kappa \frac{\partial y}{\partial \omega} \left[(1 - k_p) \beta_w + (k_w - 1) \frac{\beta_p}{y^p} \right] \\
J_{12} &= \omega_0 \kappa \frac{\partial y}{\partial \lambda} \left[(1 - k_p) \beta_w + (k_w - 1) \frac{\beta_p}{y^p} \right] \\
J_{21} &= (c - 1) \frac{\partial \omega y}{\partial \omega} - \lambda_0 \left[\frac{\partial g_k}{\partial \omega} + \kappa \frac{\partial y}{\partial \omega} \left(\frac{\beta_p}{y^p} + \kappa_p \beta_w \right) \right] \\
J_{22} &= (c - 1) \left[\frac{\partial \omega y}{\partial \lambda} - i_\lambda \right] - (\bar{\pi} + n) - \lambda_0 \left[\frac{\partial g_k}{\partial \lambda} + \kappa \frac{\partial y}{\partial \lambda} \left(\frac{\beta_p}{y^p} + \kappa_p \beta_w \right) \right]
\end{aligned}$$

with the following partial derivatives¹⁹:

$$\begin{aligned}
\frac{\partial y}{\partial \omega} &= -y_0(i_r - c)/D \\
\frac{\partial y}{\partial \lambda} &= \frac{-ci_\lambda}{D} \\
\frac{\partial \omega y}{\partial \omega} &= \frac{y_0}{D}(1 - i_r) \\
\frac{\partial g_k}{\partial \omega} &= -i_r \frac{y_0}{D}(1 - c) \\
\frac{\partial g_k}{\partial \lambda} &= -i_r(1 - \omega_0) \frac{ci_\lambda}{D} \\
\frac{\partial \omega y}{\partial \lambda} &= \frac{-\omega_0 ci}{D}
\end{aligned}$$

In order to simplify the entry J_{22} we assume that $i_\lambda = \bar{\pi} + n$. The trace ($J_{11} + J_{22}$) can be expressed as follow:

$$\begin{aligned}
\text{Trace} &= \omega_0 \kappa \frac{y_0(c - i_r)}{D} \left[(1 - k_p)\beta_w + (k_w - 1) \frac{\beta_p}{y^p} \right] + \\
&\quad ci_\lambda(1 - \omega_0)(i_r(1 + \lambda_0) - 1)/D + \lambda_0 \kappa ci_\lambda \left(\frac{\beta_p}{y^p} + \kappa_p \beta_w \right) / D
\end{aligned}$$

The Jacobian matrix can be simplified according to the following steps:

1. Factorizing $\omega_0 \frac{\kappa}{D} \left[(1 - k_p)\beta_w + (k_w - 1) \frac{\beta_p}{y^p} \right]$ in line 1
2. Factorizing $i_\lambda c$ in column 2
3. Factorizing y_0 in column 1
4. Factorizing D from line 2

$$|J| = A \begin{vmatrix} c - i_r & -1 \\ (c - 1)(1 - i_r(1 + \lambda_0)) + \lambda_0 \kappa(i_r - c)B & (1 - \omega_0)(i_r(1 + \lambda_0) - 1) + \lambda_0 \kappa B \end{vmatrix}$$

with:

¹⁹At the steady state $\frac{N_0}{D_0} = y_0$.

$$\begin{aligned}
A &= \omega_0 \frac{\kappa}{D^2} \left[(1 - k_p) \beta_w + (k_w - 1) \frac{\beta_p}{y^p} \right] i_\lambda c y_0 \\
B &= \left(\frac{\beta_p}{y^p} + \kappa_p \beta_w \right)
\end{aligned}$$

The sign of the determinant is given by:

$$|J| = \left[(1 - k_p) \beta_w + (k_w - 1) \frac{\beta_p}{y^p} \right] \left(1 - i_r (1 + \lambda_0) \right) \left(\omega_0 c + i_r (1 - \omega_0) - 1 \right)$$

The stability conditions can be summarized as follow:

$$\begin{aligned}
\text{Trace} &= \frac{(W + X)}{D} \\
|J| &= TZ(-D)
\end{aligned}$$

with

$$\begin{aligned}
W &= \omega_0 \kappa y_0 (c - i_r) T \\
X &= c i_\lambda (1 - \omega_0) (-Z) + \lambda_0 \kappa c i_\lambda \left(\frac{\beta_p}{y^p} + \kappa_p \beta_w \right) \\
D &= 1 - i_r - \omega (c - i_r) \\
Z &= 1 - i_r (1 + \lambda_0) \\
T &= \left[(1 - k_p) \beta_w + (k_w - 1) \frac{\beta_p}{y^p} \right]
\end{aligned}$$

10.6 Stability proof: credit rationing and debt

The entries of the Jacobian matrix of the system of equations 28 and 29 in section 5 are:

$$\begin{aligned}
J_{11} &= \beta_c y_{wo} + (c_0 - 1) \left[\omega_0 \frac{\partial y}{\partial \lambda} - i_\lambda \right] - (\bar{\pi} + n) - \lambda_0 \frac{\partial g_k}{\partial \lambda} \\
J_{12} &= -\beta_c y_{wo} + (c_0 - 1) \omega_0 \frac{\partial y}{\partial d} - \lambda_0 \frac{\partial g_k}{\partial d} \\
J_{21} &= \frac{\partial y}{\partial \lambda} (1 - \omega_0) + \alpha_{\pi_b} i_\lambda - \frac{\partial g_k}{\partial \lambda} (1 + d_0) \\
J_{22} &= \frac{\partial y}{\partial d} (1 - \omega_0) + i_d (1 - \alpha_{\pi_b}) - \frac{\partial g_k}{\partial d} (1 + d_0) - (\bar{\pi} + n)
\end{aligned}$$

with the following partial derivatives:

$$\begin{aligned}\frac{\partial y}{\partial \lambda} &= \frac{-i_\lambda(\lambda_0\beta_c + c_0) + \beta_c y_0 \omega_0}{D} \\ \frac{\partial y}{\partial d} &= -\frac{\beta_c}{D c_0} (y_0 - n - \delta) \\ \frac{\partial g_k}{\partial \lambda} &= i_r(1 - \omega_0) \frac{\partial y}{\partial \lambda} \\ \frac{\partial g_k}{\partial d} &= i_r(1 - \omega_0) \frac{\partial y}{\partial d}\end{aligned}$$

J_{11} and J_{22} can be re-organized as follow, assuming $i_\lambda = \bar{\pi} + n$:

$$\begin{aligned}J_{11} &= (1 - \omega_0) [1 - i_r(1 + \lambda_0)] (\beta_c y^w - c i_\lambda) / D \\ J_{22} &= -\frac{\beta_c}{D} y^w (1 - \omega_0) [1 - i_r(1 + d_0)] + (\bar{\pi} + n + \xi_\lambda) - \alpha_{\pi_b} i_d - (\bar{\pi} + n)\end{aligned}$$

The trace ($J_{11} + J_{22}$) can be expressed as follow:

$$\text{Trace} = -c i_\lambda (1 - \omega_0) [1 - i_r(1 + \lambda_0)] / D + y^w \beta_c (1 - \omega_0) i_r (d_0 - \lambda_0) / D - \xi_\lambda - \alpha_{\pi_b} i_d < 0$$

The parameter for credit rationing β_c increases the negativity of the trace if $d_0 - \lambda_0 < 0$. Substituting d_0 and λ_0 by their steady states values gives:

$$d_0 - \lambda_0 = (c - 1) y^w [(n + \bar{\pi} - i_d)(\alpha_{\pi_b} - 1)] - n \bar{\pi} - \bar{\pi}^2 < 0$$

The expression ($d_0 - \lambda_0$) is always negative as we know from the steady state values that $n + \bar{\pi} > i_d$ and that $\alpha_{\pi_b} < 1$ from the parameters of the model.

Rearranging J_{12} and J_{21} , we get:

$$\begin{aligned}J_{12} &= -\beta_c y_{wo} [(1 - \omega_0) [1 - i_r(1 + \lambda_0)]] / D < 0 \\ J_{21} &= (\beta_c y_{wo} - i_\lambda c_0) [(1 - \omega_0) [1 - i_r(1 + d_0)]] / D + \alpha_{\pi_b} i_\lambda\end{aligned}$$

The sign of the entry J_{21} is unclear. To obtain the sign of the Jacobian, the following simplifications can be made:

1. Factorizing $(1 - \omega_0)[1 - i_r(1 + \lambda_0)]/D$ from line 1
2. Factorizing $1/D$ from line 2
3. Factorizing $(\beta_c y_{wo} - i_\lambda c_0)$ from column 1
4. Factorizing $-\beta_c y_{wo}$ from column 2

The Jacobian matrix now appears as:

$$|J| = A \begin{vmatrix} 1 & 1 \\ \left[(1 - \omega_0)[1 - i_r(1 + d_0)] \right] + \frac{\alpha_{\pi_b} i_\lambda D}{(\beta_c y_{wo} - i_\lambda c_0)} & \left[(1 - \omega_0)[1 - i_r(1 + d_0)] \right] + \frac{(\alpha_{\pi_b} i_d + \xi_\lambda) D}{(\beta_c y_{wo})} \end{vmatrix}$$

with:

$$A = -\beta_c y_{wo} (\beta_c y_{wo} - i_\lambda c_0) (1 - \omega_0) [1 - i_r(1 + \lambda_0)] / D^2$$

The sign of the Jacobian is given by:

$$|J| = A \left[\frac{(\alpha_{\pi_b} i_d + \xi_\lambda)}{(\beta_c y_{wo})} - \frac{\alpha_{\pi_b} i_\lambda}{(\beta_c y_{wo} - i_\lambda c_0)} \right] > 0$$

which is always greater than zero if $\beta_c < i_\lambda c_0 / y_{wo}$.

10.7 Stability proof: debt default and income distribution

This section presents the stability properties of the system of equations 30 – 31 in section 6. The entries J_{11} and J_{12} of the Jacobian matrix are left unchanged with respect to the system of equations 26 – 27 (see section 10.5). The entries J_{21} and J_{22} are now increasing with $\beta_\varphi \lambda_0$.

$$\begin{aligned} J_{21} &= (c + \beta_\varphi \lambda_0 - 1) \frac{\partial \omega y}{\partial \omega} - \lambda_0 \left[\frac{\partial g_k}{\partial \omega} + \kappa \frac{\partial y}{\partial \omega} \left(\frac{\beta_p}{y^p} + \kappa_p \beta_w \right) \right] \\ J_{22} &= (c + \beta_\varphi \lambda_0 - 1) \left[\frac{\partial \omega y}{\partial \lambda} - i_\lambda \right] - (\bar{\pi} + n + n_\varphi) - \lambda_0 \left[\frac{\partial g_k}{\partial \lambda} + \kappa \frac{\partial y}{\partial \lambda} \left(\frac{\beta_p}{y^p} + \kappa_p \beta_w \right) \right] \end{aligned}$$

The following partial derivatives $\frac{\partial y}{\partial \omega}$, $\frac{\partial y}{\partial \lambda}$, $\frac{\partial \omega y}{\partial \omega}$, $\frac{\partial g_k}{\partial \omega}$, $\frac{\partial g_k}{\partial \lambda}$ and $\frac{\partial \omega y}{\partial \lambda}$ are left unchanged.

In order to simplify the entry J_{22} we assume that $i_\lambda = \bar{\pi} + n + n_\varphi$. The trace ($J_{11} + J_{22}$) is now decreasing with β_φ ensuring a negative trace:

$$\begin{aligned} \text{Trace} &= \omega_0 \kappa \frac{y_0(c - i_r)}{D} \left[(1 - k_p)\beta_w + (k_w - 1)\frac{\beta_p}{y^p} \right] + \\ &\quad ci_\lambda(1 - \omega_0)(i_r(1 + \lambda_0) - 1)/D - \beta_\varphi \lambda_0 i_\lambda(1 - i_r(1 - \omega_0))/D + \lambda_0 \kappa ci_\lambda \left(\frac{\beta_p}{y^p} + \kappa_p \beta_w \right) / D \end{aligned}$$

The Jacobian matrix can be simplified according to the following steps:

1. Factorizing $\omega_0 \frac{\kappa}{D} \left[(1 - k_p)\beta_w + (k_w - 1)\frac{\beta_p}{y^p} \right]$ in line 1
2. Factorizing i_λ in column 2
3. Factorizing y_0 in column 1
4. Factorizing D from line 2

$$|J| = A \begin{vmatrix} c - i_r & -c \\ (c - 1)(1 - i_r(1 + \lambda_0)) + \beta_\varphi \lambda_0(1 - i_r) + \lambda_0 \kappa(i_r - c)B & c(1 - \omega_0)(i_r(1 + \lambda_0) - 1) - \beta_\varphi \left(1 - i_r(1 - \omega_0) \right) + \lambda_0 \kappa cB \end{vmatrix}$$

with:

$$\begin{aligned} A &= \omega_0 \frac{\kappa}{D^2} \left[(1 - k_p)\beta_w + (k_w - 1)\frac{\beta_p}{y^p} \right] i_\lambda c y_0 \\ B &= \left(\frac{\beta_p}{y^p} + \kappa_p \beta_w \right) \end{aligned}$$

The sign of the determinant is given by:

$$|J| = \left[(1 - k_p)\beta_w + (k_w - 1)\frac{\beta_p}{y^p} \right] \left(\omega_0 c + i_r(1 - \omega_0) - 1 \right) \left[\left(1 - i_r(1 + \lambda_0) \right) - \beta_\varphi \lambda_0 i_r \right]$$

The stability conditions can be summarized as follow:

$$\begin{aligned} \text{Trace} &= \frac{(W + X)}{D} \\ |J| &= T(Z - \beta_\varphi \lambda_0 i_r)(-D) \end{aligned}$$

with

$$\begin{aligned}
W &= \omega_0 \kappa y_0 (c - i_r) T \\
X &= c i_\lambda (1 - \omega_0) (-Z) - \beta_\varphi \lambda_0 i_\lambda (1 - i_r (1 - \omega_0)) + \lambda_0 \kappa c i_\lambda \left(\frac{\beta_p}{y^p} + \kappa_p \beta_w \right) \\
D &= 1 - i_r - \omega (c - i_r) \\
Z &= 1 - i_r (1 + \lambda_0) \\
T &= \left[(1 - k_p) \beta_w + (k_w - 1) \frac{\beta_p}{y^p} \right]
\end{aligned}$$

10.8 Stability proof: debt default and credit rationing

The entries of the Jacobian matrix of the system of equations 32 and 33 in section 6.2 are:

$$\begin{aligned}
J_{11} &= \beta_c y_{w0} + (c_0 + \beta_\varphi \lambda_0 - 1) \left[\omega_0 \frac{\partial y}{\partial \lambda} - i_\lambda \right] - (\bar{\pi} + n + n_\varphi) - \lambda_0 \frac{\partial g_k}{\partial \lambda} \\
J_{12} &= -\beta_c y_{w0} + (c_0 + \beta_\varphi \lambda_0 - 1) \omega_0 \frac{\partial y}{\partial d} - \lambda_0 \frac{\partial g_k}{\partial d}
\end{aligned}$$

with the following partial derivatives:

$$\begin{aligned}
\frac{\partial y}{\partial \lambda} &= \frac{-i_\lambda c_0 + \beta_c y_{w0}}{D} \\
\frac{\partial y}{\partial d} &= -\frac{\beta_c y_{w0}}{D}
\end{aligned}$$

J_{21} and J_{22} as well as $\frac{\partial y}{\partial \lambda}$, $\frac{\partial y}{\partial d}$, $\frac{\partial g_k}{\partial \lambda}$ and $\frac{\partial g_k}{\partial d}$ are similar to the previous section (section 5).

J_{11} and J_{22} can be re-organized as follow, assuming $i_\lambda = \bar{\pi} + n + n_\varphi$:

$$\begin{aligned}
J_{11} &= \frac{1}{D} \left[(1 - \omega_0) \left[1 - i_r (1 + \lambda_0) \right] \left(\beta_c y^w - c i_\lambda \right) - \beta_\varphi \lambda_0 \left[i_\lambda \left[1 - i_r (1 - \omega_0) \right] - \omega_0 y_{w0} \beta_c \right] \right] \\
J_{22} &= -\frac{\beta_c}{D} y^w (1 - \omega_0) \left[1 - i_r (1 + d_0) \right] - \alpha_{\pi_b} i_d + n_\varphi - \xi_\lambda
\end{aligned}$$

The trace ($J_{11} + J_{22}$) can be expressed as follow:

$$\begin{aligned}
\text{Trace} &= -c i_\lambda (1 - \omega_0) \left[1 - i_r (1 + \lambda_0) \right] / D - y_{w0} \beta_c (1 - \omega_0) i_r (\lambda_0 - d_0) / D \\
&\quad - \beta_\varphi \lambda_0 \left[i_\lambda \left[1 - i_r (1 - \omega_0) \right] - \omega_0 y_{w0} \beta_c \right] - \alpha_{\pi_b} i_d + n_\varphi - \xi_\lambda < 0
\end{aligned}$$

The first two elements of the trace are similar to that of previous section. They are both negative and the parameter for credit rationing β_c increases the negativity of the second element, given that banks have a positive net wealth at the steady state $\lambda_0 - d_0 > 0$.

Rearranging J_{12} and J_{21} , we obtain:

$$\begin{aligned} J_{12} &= -\beta_c y_{wo} \left[(1 - \omega_0)[1 - i_r(1 + \lambda_0)] \right] / D - \beta_c \omega_0 y_{wo} \lambda_0 \beta_\varphi / D < 0 \\ J_{21} &= (\beta_c y_{wo} - i_\lambda c_0) \left[(1 - \omega_0)[1 - i_r(1 + d_0)] \right] / D + \alpha_{\pi_b} i_\lambda \end{aligned}$$

with the entry J_{21} being similar to the previous section. The sign of the entry J_{21} is unclear. To get the sign of the Jacobian, the following simplifications can be undertaken:

1. Factorizing $1/D$ from line 1 and line 2
2. Factorizing $(\beta_c y_{wo} - i_\lambda c_0)$ from column 1
3. Factorizing $-\beta_c y_{wo}$ from column 2
4. Factorizing $(1 - \omega_0) \left[1 - i_r(1 + d_0) \right]$ from line 2

Assuming in addition that the rate of debt default in the long term n_φ is equal to the mark up of banks in the long term ξ_λ (meaning that debt defaults are compensated by banks profits at the steady state); assuming further that profits of banks distributed to capitalists α_{π_b} are small. The Jacobian matrix now appears as:

$$|J| = A \begin{vmatrix} \left[(1 - \omega_0)[1 - i_r(1 + \lambda_0)] \right] - \beta_\varphi \lambda_0 \left[i_\lambda \left[1 - i_r(1 - \omega_0) \right] - \omega_0 y_{wo} \beta_c \right] & \left[(1 - \omega_0)[1 - i_r(1 + \lambda_0)] \right] + \beta_c \lambda_0 \beta_\varphi \omega_0 \\ 1 & 1 \end{vmatrix}$$

with:

$$A = -\beta_c y_{wo} (\beta_c y_{wo} - i_\lambda c_0) (1 - \omega_0) \left[1 - i_r(1 + d_0) \right] / D^2$$

The sign of the Jacobian is given by:

$$\begin{aligned} |J| &= A \left[-\frac{\beta_\varphi \lambda_0}{(\beta_c y_{wo} - i_\lambda c_0)} \left[i_\lambda \left[1 - i_r(1 - \omega_0) \right] \right] \right. \\ &\quad \left. - \beta_c \lambda_0 \beta_\varphi \omega \left[1 - \frac{y_{wo}}{(\beta_c y_{wo} - i_\lambda c_0)} \right] \right] \end{aligned}$$

The determinant is likely to be positive given that A is positive and that $-\frac{\beta_\varphi \lambda_0}{(\beta_c y_{wo} - i_\lambda c_0)} \left[i_\lambda \left[1 - i_r(1 - \omega_0) \right] \right]$ is positive too to the extent that credit rationing is small $\beta_c < i_\lambda c_0 / y_{wo}$. The element $-\beta_c \lambda_0 \beta_\varphi \omega \left[1 - \frac{y_{wo}}{(\beta_c y_{wo} - i_\lambda c_0)} \right]$ is negative but this effect is small if debt default is limited: β_φ is small.

10.9 Stability proof: endogenous mark up - no credit rationing $\beta_c = 0$

$$\begin{aligned} J_{11} &= (c_0 - 1) \left[\omega_0 \frac{\partial y}{\partial \lambda} - i_0^\lambda - \lambda_0 \frac{\partial i_\lambda}{\partial \lambda} \right] - (\bar{\pi} + n) - \lambda_0 \frac{\partial g_k}{\partial \lambda} \\ J_{12} &= (c_0 - 1) \left[\omega_0 \frac{\partial y}{\partial d} - \lambda_0 \frac{\partial i_\lambda}{\partial d_0} \right] - \lambda_0 \frac{\partial g_k}{\partial d} \\ J_{21} &= \frac{\partial y}{\partial \lambda} (1 - \omega_0) + \alpha_{\pi_b} \left[i_\lambda + \lambda_0 \frac{\partial i_\lambda}{\partial \lambda} \right] - \frac{\partial g_k}{\partial \lambda} (1 + d_0) \\ J_{22} &= \frac{\partial y}{\partial d} (1 - \omega_0) + i_d + \alpha_{\pi_b} \left[\lambda_0 \frac{\partial i_\lambda}{\partial d} - i_d \right] - \frac{\partial g_k}{\partial d} (1 + d_0) - (\bar{\pi} + n) \end{aligned}$$

with the following partial derivatives:

$$\begin{aligned} \frac{\partial y}{\partial \lambda} &= -\frac{c_0}{D} \left[i_\lambda - \beta_{bis} d_0 / \lambda_0 \right] \\ \frac{\partial i_\lambda}{\partial \lambda} &= -\beta_{bis} d_0 / \lambda_0^2 \\ \frac{\partial y}{\partial d} &= -\frac{c_0}{D} \beta_{bis} \\ \frac{\partial i_\lambda}{\partial d} &= \beta_{bis} / \lambda_0 \\ \frac{\partial g_k}{\partial \lambda} &= i_r (1 - \omega_0) \frac{\partial y}{\partial \lambda} \\ \frac{\partial g_k}{\partial d} &= i_r (1 - \omega_0) \frac{\partial y}{\partial d} \end{aligned}$$

J_{11} and J_{22} can be expressed as follows:

$$\begin{aligned} J_{11} &= \left(-i_\lambda + \beta_{bis} \frac{d_0}{\lambda_0} \right) \left[c_0 - 1 - i_r (1 - \omega_0) (c_0 (1 + \lambda_0) - 1) \right] - (\bar{\pi} + n) \\ J_{22} &= -\frac{c_0}{D} \beta_{bis} (1 - \omega_0) \left[1 - (1 + d_0) i_r \right] - \xi_\lambda + \alpha_{\pi_b} (\beta_{bis} - i_d) \end{aligned}$$

J_{11} is negative if two conditions hold. The mark up is rigid $\beta_{bis} < \frac{i_\lambda \lambda}{d_0}$ and the propensity to invest is small $i_r < \frac{c_0 - 1}{(1 - \omega_0) [c_0 (1 + \lambda_0) - 1]}$. J_{22} is negative too as we know from Proposition 4 that α_{π_b} is small. The entry J_{21} and J_{12} can be re-arranged as follows:

$$\begin{aligned}
J_{21} &= -\frac{c_0}{D} \left(i_\lambda - \beta_{bis} \frac{d_0}{\lambda_0} \right) (1 - \omega_0) \left[1 - (1 + d_0) i_r \right] + \alpha_{\pi_b} \left(i_\lambda - \beta_{bis} \frac{d_0}{\lambda_0} \right) \\
J_{12} &= -\beta_{bis} \left[c_0 - 1 - i_r (1 - \omega_0) \left(c_0 (1 + \lambda_0) - 1 \right) \right]
\end{aligned}$$

J_{21} is negative given the assumptions made above about the values of β_{bis} and α_{π_b} . Similarly, J_{12} is always negative given the assumption on the value of i_r made previously. The sign of the determinant is therefore ambiguous. A simple analytic solution can be obtained by making the following simplifications and assumptions:

1. Factorizing $1/D > 0$ from line 1 and line 2
2. Factorizing $A = \left[c_0 - 1 - i_r (1 - \omega_0) \left(c_0 (1 + \lambda_0) - 1 \right) \right] > 0$ from ligne 1
3. Factorizing $B = -(i_\lambda - \beta_{bis} \frac{d_0}{\lambda_0}) < 0$ from column 1
4. Factorizing $C = -c_0 (1 - \omega_0) \left[1 - i_r (1 + d_0) \right] < 0$ from line 2
5. Factorizing β_{bis} from column 2
6. Assuming that α_{π_b} is close to zero.

$$|J| = E \begin{vmatrix} 1 - \frac{\bar{\pi} + n}{ABD} & -1 \\ -1 & 1 - \frac{\xi_\lambda}{DC} \end{vmatrix}$$

with:

$$E = \frac{ABC\beta_{bis}}{D^2} > 0$$

The sign of the Jacobian is given by:

$$|J| = E \left[\left(1 - \frac{\bar{\pi} + n}{ABD} \right) \left(1 - \frac{\xi_\lambda}{DC} \right) - 1 \right]$$

which is positive given that $-\frac{\bar{\pi} + n}{ABD} > 0$ and that $-\frac{\xi_\lambda}{DC} > 0$.

References

- Barba, A. and M. Pivetti (2009). Rising household debt: its causes and macroeconomic implications - a long-period analysis. *Cambridge Journal of Economics* 33(113-137).
- Barbosa-Filho, N. H. and L. Taylor (2006). Distributive and demand cycles in the us economy-a structuralist goodwin model. *Metroeconomica* 57(3), 389–411.
- Bhaduri, A. and S. Marglin (1990). Unemployment and the real wage: the economic basis for contesting political ideologies. *Cambridge Journal of Economics* 14(4), 375–93.
- Charpe, M., C. Chiarella, P. Flaschel, and W. Semmler (2011). *Financial Assets, Debt and Liquidity Crises*. Cambridge University Press.
- Charpe, M., P. Flaschel, C. Proano, and W. Semmler (2009). Overconsumption, credit rationing and bailout monetary policy: A minskyan perspective. *Intervention: European Journal of Economics and Economic Policies* 6(02).
- Chiarella, C., P. Flaschel, and Franke (2005). *Foundations for a Disequilibrium Theory of the Business Cycle*. Cambridge University Press.
- Chiarella, C., P. Flaschel, G. Gong, and W. Semmler (2003). Non linear phillips curves, complex dynamics and monetary policy in a keynesian macro model. *Chaos, Solution and Fractals* 18, 613 – 634.
- Crotty, J. (2009). Structural causes of the global financial crisis: a critical assessment of the "new financial architecture". *Cambridge Journal of Economics* 33, 563–580.
- Dos Santos, C. and G. Zezza (2005). A simplified stock-flow consistent post-keynesian growth model. *The Levy Economic Institute Working Papers* (421).
- Duménil, G. and D. Levy (1999). Being keynesian in the short term and classical in the long term. *The Manchester School* 67(6), 621–740.
- Dutt, A. K. (2006). Maturity, stagnation and consumer debt: A steindlian approach. *Metroeconomica* 57(3)(339-364).
- Goodwin, R. M. (1967). *A growth cycle*. in *Socialism, Capitalism and Economic Growth*.
- Lavoie, M. and W. Godley (2007). *Monetary Economics: An Integrated Approach to Credit, Money, Income, Production and Wealth*. Palgrave/Macmillan. 448 p.

- Le Heron, E. and T. Mouakil (2008). A post-keynesian stock-flow consistent model for dynamic analysis of monetary policy shock on banking behaviour. *Metroeconomica* 59-3, 405–440.
- Palma, J. G. (2009). The revenge of the market on the rentiers. why neo-liberal reports of the end of history turned out to be premature. *Cambridge Journal of Economics* 33, 829–869.
- Pollin, R. (1988). The growth of us household debt: demand-side influences. *Journal of Macroeconomics* 10(2), 231–48.
- Proaño, C. R., P. Flaschel, H.-M. Krolzig, and M. B. Diallo (2010). Monetary policy and macroeconomic stability under alternative demand regimes. *Cambridge Journal of Economics* 1(1-17).
- Rose, H. (1967). On the nonlinear theory of the employment cycle. *Review of Economic Studies* 34(2), 153–173.
- Wray, R. L. (2009). The rise and fall of money manager capitalism: a minskian approach. *Cambridge Journal of Economics* 33, 807–828.

Publisher: Hans-Böckler-Stiftung, Hans-Böckler-Str. 39, 40476 Düsseldorf, Germany
Phone: +49-211-7778-331, IMK@boeckler.de, <http://www.imk-boeckler.de>

IMK Working Paper is an online publication series available at:
<http://www.boeckler.de/cps/rde/xchg/hbs/hs.xls/31939.html>

ISSN: 1861-2199

The views expressed in this paper do not necessarily reflect those of the IMK or the Hans-Böckler-Foundation.

All rights reserved. Reproduction for educational and non-commercial purposes is permitted provided that the source is acknowledged.

**Hans Böckler
Stiftung** 

Fakten für eine faire Arbeitswelt.
