IMK
Institut für Makroökonomie
und Konjunkturforschung
Macroeconomic Policy Institute

November 2013

127

Working Paper

Daniele Tavani [1] and Luca Zamparelli [2]

Endogenous Technical Change, Employment and Distribution in the Goodwin Model

Abstract

The Goodwin (1967) model of the growth cycle assigns distributional conflict a central role in the dynamics of capital accumulation, but is silent on the determinants of technical change. Following Shah and Desai (1981), previous studies focused on the effects of the direction, or bias of technical change on the growth cycle (van der Ploeg, 1987; Foley, 2003; Julius, 2005). Either implicitly or explicitly, these contributions adopted the induced innovation hypothesis by Kennedy (1964): there exists an innovation possibility frontier out of which profit-maximizing firms freely choose the optimal combination of capital– and labor–augmenting technical change, without having to allocate resources to R&D. Our focus is instead on the choice of intensity of technical change, that is the share of R&D expenditure in output. In our framework, innovation is a costly, forward–looking process financed out of profits, and pursued by owners of capital stock (capitalists) in order to foster labor productivity and save on labor requirements. Our main findings are: (i) similarly to the literature on the direction of technical change, an endogenous intensity of R&D ultimately dampens the distributive cycle; however, (ii) steady state per capita growth, income distribution and employment rate are endogenous, and depend on the capitalists' discount rate, the institutional variables regulating the labor market, and the size of subsidies to R&D activity. Implementing the model numerically, we show that: (iii) a reduction in the capitalists' discount rate lowers per–capita growth, the employment rate and the labor share; (iv) an increase in workers' bargaining power raises the labor share, while reducing employment and per–capita growth; (v) a balanced budget increase in the R&D subsidy also fosters per-capita growth, at the expenses of the labor share. The variations corresponding to (iv) and (v), however, can be small.

1 Daniele Tavani, Colorado State University, Fort Collins, CO, US; E-Mail: daniele.tavani@colostate.edu.
2 Luca Zamparelli, La Sapienza University of Rome, Italy; E-Mail: luca.zamparelli@uniroma1.it.

Endogenous Technical Change, Employment and Distribution in the Goodwin Model

Daniele Tavani, Colorado State University, Fort Collins, CO, US
Luca Zamparelli, La Sapienza University of Rome, Italy*

November 12, 2013

Abstract

The Goodwin (1967) model of the growth cycle assigns distributional conflict a central role in the dynamics of capital accumulation, but is silent on the determinants of technical change. Following Shah and Desai (1981), previous studies focused on the effects of the *direction*, or *bias* of technical change on the growth cycle (van der Ploeg, 1987; Foley, 2003; Julius, 2005). Either implicitly of explicitly, these contributions adopted the induced innovation hypothesis by Kennedy (1964): there exists an *innovation possibility frontier* out of which profit-maximizing firms *freely* choose the optimal combination of capital– and labor–augmenting technical change, without having to allocate resources to R&D. Our focus is instead on the choice of *intensity* of technical change, that is the share of R&D expenditure in output. In our framework, innovation is a costly, forward–looking process financed out of profits, and pursued by owners of capital stock (capitalists) in order to foster labor productivity and save on labor requirements. Our main findings are: (i) similarly to the literature on the direction of technical change, an endogenous intensity of R&D ultimately dampens the distributive cycle; however, (ii) steady state per capita growth, income distribution and employment rate are endogenous, and depend on the capitalists' discount rate, the institutional variables regulating the labor market, and the size of subsidies to R&D activity. Implementing the model numerically, we show that: (iii) a reduction in the capitalists' discount rate lowers per–capita growth, the employment rate and the labor share; (iv) an increase in workers' bargaining power raises the labor share, while reducing employment and per–capita growth; (v) a balanced budget increase in the R&D subsidy also fosters per–capita growth, at the expenses of the labor share. The variations corresponding to (iv) and (v), however, can be small.

Keywords: Endogenous Technical Change, Goodwin Model, Income Shares, Employment.
JEL Codes: E32, O33

*Tavani: Department of Economics, 1771 Campus Delivery, Fort Collins, CO 80523-1771. Email: daniele.tavani@colostate.edu. Zamparelli: Dipartimento di Scienze Sociali ed Economiche, p. le Aldo Moro 5, Rome, Italy 00185. Email: luca.zamparelli@uniroma1.it. We thank Corrado Di Guilmi, Duncan Foley, Giancarlo Gandolfo, Codrina Rada, Peter Skott, Rick van der Ploeg, and participants at the Eastern Economic Association Conference 2013 and at the FMM conference 2013 for very useful comments on earlier drafts. The usual disclaimer applies.

1 Introduction

The celebrated paper by Goodwin (1967) puts distributional conflict at the heart of the growth process. Accumulation is driven by savings out of profits, and therefore is class–based. The byproduct of investment in new capital goods by asset–owners (capitalists, in typical two–class modeling jargon) is an increase in firms' demand for labor, which in turn puts upward pressure on real wages relative to labor productivity, thus increasing the share of wages in output. Now that workers have gained distributional ground, profitability suffers, and accumulation slows down. Employment will recede, and real wages will fall relative to labor productivity. At this point, profitability is restored, and accumulation can pick up again. In contrast with the neoclassical growth model, where the labor force is continuously fully employed and distributive shares evolve monotonically to their steady state value,[1] the Goodwin model sees employment and the labor share to endlessly cycle along an exponential path given by population and productivity growth. Thus, the distributional conflict that determines the Goodwin growth cycles is never settled.

Partly responsible for the outcome of perpetual conflict over income distribution is the assumption of exogenous technical change made originally by Goodwin. Following the seminal paper by Shah and Desai (1981), several studies focused on the effects of the *direction*, or *bias* of technical change on the growth cycle (van der Ploeg, 1987; Foley, 2003; Julius, 2005). Either implicitly or explicitly, these contributions adopted the induced innovation hypothesis put forward by Kennedy (1964). The hypothesis postulates the existence of an *innovation possibility frontier*, out of which profit-maximizing firms *freely* choose the optimal combination of capital– and labor–augmenting technical change, without having to allocate resources to R&D. In this framework, labor productivity growth turns out to be directly related to the labor share. Thus, labor–augmenting technical change is *induced*, or cost–driven: the higher the labor share in output (costs), the stronger the bias of technical progress toward labor–augmenting technologies. The introduction of a feedback from income distribution to labor productivity changes the dynamics of the interaction between employment and distribution: the steady state equilibrium becomes locally stable. As noted by Shah and Desai (1981, p.1008), induced technical change provides capitalists with an additional 'weapon', other than reducing capital accumulation, to confront workers' demands. The new instrument allows them to break the symmetric bargaining positions, so that the growth cycle vanishes in the long–run. While cycles are not persistent, however, they do not disappear. Before they converge to the long–run position of the economy, the labor share and the employment rate cycle in typical predator–prey fashion, the employment rate behaving like a prey and the labor share behaving like a predator. The oscillations become smaller and smaller, until negligible, as is the case in Foley (2003).

Our starting point is the observation that, according to the literature on induced innovation, using the direction of technical change in order to get an advantage in the conflict over income distribution comes at no cost to the owners of capital stock. Already Nordhaus (1971) expressed skepticism about models based on induced innovation, because they lacked an account of the resource constraints faced by asset–owners in innovating to save on labor requirements. To overcome this criticism, in this paper we borrow from the standard endogenous growth literature (see Aghion and Howitt, 2010, for a summary) to introduce endogenous, labor–augmenting technological change in the Goodwin (1967) model. In our framework, innovation is a costly, forward–looking process

[1] The labor share grows, is constant, or decreases during the transition to the steady state depending on whether the elasticity of substitution between capital and labor is smaller, equal, or larger than one.

financed out of profits, and pursued by firms in order to foster labor productivity. Thus, there is no automatic positive feedback from increases in the labor share on productivity growth. In particular, extending the basic setup presented by Foley and Michl (1999), we model forward-looking capitalist households choosing the time path of R&D and physical capital investment in order to maximize a measure of their intertemporal consumption. Given per–period profits, there are trade–offs between accumulating new physical capital and investing in R&D. These trade–offs are absent in the induced innovation literature, in which innovation intensity is given or, more appropriately, costless.

The purpose of adding endogenous growth elements to the Goodwin model is twofold. First, our aim is to investigate what is the impact of the costly nature of labor augmenting innovation on the time path followed by the stylized economy that the model is meant to describe. In this respect, our framework produces a hybrid between mainstream endogenous growth models, in which optimization by forward–looking agents is typically characterized by saddle–path stability, and the dampened employment–distribution oscillations one finds for instance in Foley (2003). On the one hand, our reduced–form dynamical system fulfills the well–known Blanchard and Kahn (1980) (or Gandolfo, 1997, Chapters 18 and 22) requirements for *conditional stability*: the Jacobian matrix –which summarizes the approximate behavior of the system around its steady state– has as many unstable eigenvalues as there are forward–looking variables. The forward–looking variables in the model are capitalist consumption and R&D spending, both normalized by the level of technology; they are free to 'jump' onto the stable manifold that ensures convergence to the steady state. On the other hand, the stable eigenvalues are complex conjugate with negative real parts. For this reason, once initial conditions are picked on the forward looking variables, convergence to the steady state will occur in cyclical fashion, similarly to the induced innovation literature.[2]

Second, and perhaps more important, by making technical progress the outcome of utility maximizing capitalist households the steady state values of productivity growth, income distribution and employment become endogenous. They depend on: (a) capitalists' intertemporal preferences for consumption –which determine their propensity to save out of profits; (b) the institutional variables affecting workers' bargaining power; and (c) policy variables related to R&D incentives/subsidies. In particular, in a calibrated version of the model, we find that: an increase in the discount rate lowers per-capita growth, the employment rate and the labor share; an increase in workers' bargaining strength raises the labor share at the cost of lower employment and per-capita growth; an increase in the R&D subsidy fosters per-capita growth at the expenses of the labor share.

The rest of the paper is organized as follows. Section 2 outlines the model; Section 3 studies the associated dynamical system by characterizing its steady state, carrying out comparative dynamics exercises, analyzing its transitional dynamics, and carrying a sensitivity analysis with respect to crucial parameters. We discuss our findings in relation to the induced innovation literature in Section 3.3. Section 4 concludes.

2 The Model

2.1 Production, Income Shares, and Employment

The final good Y is produced using labor L and homogeneous capital K in fixed proportions. Time is continuous, and we rule out population growth for simplicity. Capital stock depreciates at a

[2] As explained later, however, there are important differences in the implied behavior of the economy relative to the induced innovation model.

rate $\delta > 0$ per period. Letting A denote the current state of knowledge on labor–augmenting technologies (in turn equal to labor productivity), and denoting the constant output/capital ratio by B, the production technique is:[3]

$$Y[t] = \min\{A[t]L[t], BK[t]\} \tag{1}$$

Profit maximization requires not allowing productive factors to remain idle, so that $A[t]L[t] = BK[t]\,\forall t$. It follows that total profit income in the economy is $\Pi[t] = BK[t]\left(1 - \frac{w[t]}{A[t]}\right)$. We denote the share of labor in output by $\omega[t] \equiv w[t]L[t]/Y[t] = w[t]/A[t]$, so that the profit rate $r[t] \equiv \Pi[t]/K[t] = B(1-\omega(t))$. Further, the size of the population is normalized to one, so that the employment rate of labor in the economy at time t is $v[t] = L[t] = BK[t]/A[t]$. Finally, notice that a higher knowledge base A lowers the amount of labor required in production everything else equal, so that technical change is labor–saving other than labor–augmenting.[4]

2.2 Accumulation and Innovation

We turn now to describing how resources are allocated to accumulation of new capital outlays and to innovation producing new labor–augmenting blueprints. Both types of investment raise total profits but in different ways: while capital accumulation increases the size of a firm's business, innovation reduces unit labor cost production. For this reason, the way profits are allocated to their alternative uses will depend on wage dynamics.

As it is typical in two class–models (Foley and Michl, 1999), we suppose that workers consume all of their wages, and that both innovation and accumulation are financed out of profit incomes earned by asset–owners (capitalists).[5] In order to determine how capitalists choose the allocation of their expenditure on investment, we need to specify how improvements in the state of knowledge are produced. The endogenous growth literature generally considers the flow of newly produced technologies $\dot{A}$ to depend positively on R&D inputs (either researchers or physical output investment), and on the existing level of technology itself. Accordingly, we assume

$$\dot{A}[t] = F[R[t], A[t]],$$

where R is the amount of resources invested in R&D. Considering that current research builds on past R&D experience, the productivity of a single unit of R&D investment rises with the level reached by the firm-specific knowledge. Since we are interested in studying balanced growth paths (BGPs), we impose restrictions on F that ensure their existence. Equation (1) shows that output and productivity along a BGP need be growing at the same rate; the same will be true regarding output uses (consumption, investment in physical capital, and investment in R&D). Accordingly, the R&D *intensity*, i.e. the ratio $n \equiv R/A$, needs to be constant at a BGP. We can satisfy this requirement by assuming that F is linearly homogenous. Finally, postulating a constant elasticity

[3]To minimize notational confusion, throughout this paper we denote functional dependence by square brackets, while we use round brackets only for multiplication purposes. For instance, $g[x]$ means that *g is a function of x*, while $\beta(x-q)$ means that β multiplies the difference $x-q$.

[4]This follows from the Leontief specification of the production function. However, the same feature would hold with a smooth production function with capital/labor substitution, as long as the elasticity of substitution between capital and labor is below 1.

[5]Goodwin (1967) assumed that 100% of profits were used for accumulation purposes every period, although it is easy to show that its conclusions are not sensitive to this assumption.

of innovation to R&D intensity we have $\dot{A}[t] = an[t]^{\chi}A[t]$, where $\chi \in (0,1)$ and a is a positive constant. In what follows, we denote $an[t]^{\chi} \equiv \phi[n[t]]$.

The endogenous growth literature has stressed the non–rival nature of the knowledge base A, and the related issue of potential lack of incentives to its production. While the problem of resource allocation when the outcome of R&D expenditure is not fully appropriable is an important one, our focus is on the incentives to innovate arising in the struggle over income distribution. Thus, in order to keep the class structure of the economy as close as possible to that of the Goodwin (1967) model, we assume that improvements in technology are built in–house and remain private knowledge with no possibility of being adopted by competing firms. Since there are no across-firm technological spill-overs, every firm (capitalist) has to perform its own share of R&D investment.[6]

Further, there is a government that proportionally taxes c, capitalist consumption, at a rate $\tau \in (0,1)$ in order to finance a proportional R&D subsidy $s \in (0,1)$. Imposing for simplicity a balanced government budget, we have that $\tau c[t] = sR[t]$ for all t. Once innovation expenditure R has been chosen at each moment in time, resources that are available to accumulate capital stock are total profits Π minus the sum of: depreciation δK, R&D spending $R(1-s)$, and capitalist consumption $c(1+\tau)$. Hence, the transition equation for capital stock at time t is $\dot{K}[t] = \Pi[t] - \delta K[t] - c[t](1+\tau) - R[t](1-s)$. Assuming that asset–owners have logarithmic preferences over consumption streams and discount the future at a rate $\rho > 0$, they choose sequences of consumption c and R&D spending R to solve the following optimal control problem:

$$\begin{array}{lrl} \text{Choose } \{c[t], R[t]\}_{t\in[0,\infty)} & \text{to max} & \displaystyle\int_0^{\infty} \exp\{-\rho t\} \ln c[t] dt \\ & \text{s. t.} & \dot{A}[t] = \phi[n[t]]A[t] \\ & & \dot{K}[t] = B\left(1 - \frac{w[t]}{A[t]}\right)K[t] - \delta K[t] - c[t](1+\tau) - R[t](1-s) \\ & \textit{given} & (K[0], A[0]) \equiv (K_0, A_0) > (0,0), \\ & & \lim\limits_{t\to\infty} e^{-\rho t}K[t] \geq 0 \\ & & \lim\limits_{t\to\infty} e^{-\rho t}A[t] \geq 0. \end{array} \tag{2}$$

The solution, presented in the Appendix, leads to the following laws of motion for consumption and R&D intensity along an optimal control:

$$\frac{\dot{c}}{c} = B(1-\omega) - (\rho + \delta) \tag{3}$$

$$(1-\chi)\frac{\dot{n}}{n} = B(1-\omega) - \delta - \phi'[n]\frac{\omega v}{1-s} - \big(\phi[n] - \phi'[n]n\big) \tag{4}$$

Equation (3) is the typical Euler equation for consumption: if the profit rate on capital stock $B(1-\omega)$ exceeds the discount rate (taking depreciation into account), current savings will increase leading the way to higher future consumption. On the other hand, R&D intensity growth depends negatively on the labor share because a lower profit rate reduces the general return to investment, and to R&D investment in particular.

[6]Smulders and van de Klundert (1995) rationalize this assumption with the introduction of a differentiated good, with each variety requiring specific knowledge. In their framework, product variety is functional to analyze the relation between growth and product market concentration. Since we are not concerned with similar issues we simply assume a homogeneous good.

2.3 Evolution of Employment, Factor Shares, and Productivity–adjusted Consumption

Define $d \equiv c/A$, a measure of capitalist consumption normalized by the current level of technology, or productivity–adjusted capitalist consumption. Logarithmic differentiation of the employment rate, using the balanced government budget requirement, gives:

$$\frac{\dot{v}}{v} = \frac{\dot{K}}{K} - \frac{\dot{A}}{A} = B\left(1 - \omega - \frac{(d+n)}{v}\right) - \delta - \phi[n]. \tag{5}$$

Because employment depends on the pace of accumulation and therefore on the profit share, its growth rate is lower the higher the share of labor and the higher capitalists' consumption. On the other hand, higher R&D intensity lowers labor demand for two reasons. On the one hand, it subtracts resources to the accumulation of capital. On the other hand, it increases the flow of new labor–saving discoveries, thus lowering labor demand everything else equal.

Completing the model requires to specify the dynamics of the labor share and of productivity–adjusted consumption. The tradition stemming from Goodwin (1967) assumes wage growth to depend on the employment rate as a measure of labor market tightness. In particular, $\dot{w}/w = f[v]$ such that $f' > 0$, $f[0] < 0$, and $\lim_{v \to 1} f[v] = \infty$.[7] Then, using the law of motion for labor–augmenting technologies, the labor share evolves over time according to:

$$\dot{\omega} = \left(f[v] - \phi[n]\right)\omega. \tag{6}$$

Equations (5) and (6) are the building blocks of the original Goodwin (1967) dynamical system, here modified to take into account the endogenous evolution of labor productivity and capitalist's consumption. A tighter labor market where employment is growing generates higher real wage growth, which in turn increases the share of labor. Conversely, higher R&D intensity determines a higher flow of new labor–augmenting technologies and therefore saving on labor requirements, in turn lowering the employment rate and the labor share. Because the pace of accumulation positively affects employment and thus the labor share, while innovation negatively affects both, we see that R&D effort is used to mitigate the effects of capital accumulation on workers' wage demands. This is true in our framework even without the direct feedback of the labor share on productivity growth that results from induced innovation (Shah and Desai, 1981; Foley, 2003; Julius, 2005).

Finally, using (3) and the production function of innovation, we obtain the equation describing the evolution of consumption in units of technology:

$$\frac{\dot{d}}{d} = \frac{\dot{c}}{c} - \frac{\dot{A}}{A} = B(1-\omega) - (\rho + \delta) - \phi[n]. \tag{7}$$

3 The Dynamical System

Equations (7), (4), (6), and (5) form a dynamical system describing the evolution of productivity–adjusted capitalist consumption, R&D intensity, employment, and distributive shares in the economy. We first characterize the steady state, and then we study numerically its comparative dynamics and stability properties.

[7]In using the Goodwin specification of the evolution of the labor share, we (somewhat realistically) assume that the individual capitalist household has no control over the aggregate employment effects of her decision making, and therefore does not incorporate equation (6) into its own optimization problem.

3.1 Steady State and Parameter Calibration

To get started in characterizing the steady state, consider that equation (6) gives the following *isocline* featuring the employment rate as an increasing function of R&D intensity:

$$v_{ss} = f^{-1}\left[\phi[n_{ss}]\right] \equiv h[n_{ss}] \tag{V}$$

with $\partial f^{-1}/\partial n > 0$. Higher R&D intensity produces higher labor productivity growth. If the labor share is to remain constant, higher wage growth and, in turn, a higher employment rate are required.

Next, the remaining three equations evaluated at the steady state yield conditions on the profit share that keep capitalist consumption, R&D investment and employment constant:

$$B(1-\omega_{ss}) - \delta = \rho + \phi[n_{ss}] \tag{Ω}$$

$$B(1-\omega_{ss}) - \delta = \phi'[n_{ss}]\frac{\omega_{ss}v_{ss}}{1-s} + \big(\phi[n_{ss}] - \phi'[n_{ss}]n_{ss}\big) \tag{N}$$

$$B(1-\omega_{ss}) - \delta = \phi[n_{ss}] + \frac{B(d_{ss}+n_{ss})}{v_{ss}}. \tag{D}$$

Equating the right hand sides of (Ω) and (N), and using (V) and (Ω) to eliminate respectively v_{ss} and ω_{ss}, provides the steady state value of R&D intensity as a solution to

$$\rho = \phi'[n_{ss}]\frac{h[n_{ss}](1 - B^{-1}(\rho+\delta+\phi[n_{ss}]))}{1-s} - \phi'[n_{ss}]n_{ss}. \tag{R}$$

Once the steady state R&D intensity is established, the labor share, the employment rate and capitalists' consumption in units of technology can be found as mere functions of n_{ss}: $v_{ss} = h[n_{ss}]$, $\omega_{ss} = 1 - B^{-1}((\rho+\delta) - \phi[n_{ss}])$ from (Ω), and $d_{ss} = B^{-1}\rho v_{ss} - n_{ss} = B^{-1}\rho h[n_{ss}] - n_{ss}$ from (Ω) and (D).

Plotting the isoclines (Ω) and (N) (after substituting the employment rate from V) in the (ω, n) plane provides a convenient way to characterize the interaction between distribution and innovation at the steady state and to analyze its response to parametric changes. The (Ω) isocline is unambiguously downward sloping since $d\omega/dn = -B^{-1}\phi'[n] < 0$; whereas (N) can *a priori* slope either upward or downward.[8]

The strong non–linearities that characterize the dynamical system do not allow for a closed–form solution. Thus, we implement the model numerically. Our steady state values are meant to match first unconditional moments of the relevant variables in post–WWII United States: a long–run labor share in GDP ω_{ss} around 67%, a long–run employment rate v_{ss} of about 94%, and a share of R&D in output in the neighborhood of 3%, as documented by Impullitti (2010). This latter value translates into a long–run R&D intensity $n_{ss} = (R/Y)_{ss}(Y/A)_{ss} = (R/Y)_{ss}v_{ss} = .03 \times .94 = .0282$. Accordingly, we first 'externally' calibrate a set of parameters following well–established literature. In particular, we set the discount rate at the 7% value proposed in Mehra and Prescott (1985) to match the average annual return in the US stock market. Further, we fix $\chi = .15$, within the range proposed by Kortum (1993), and we pin the innovation subsidy at 4.4%, which is the share of R&D subsidies in GDP found by Impullitti (2010) for the US. Next, we pick a standard value

[8] See the Appendix for a discussion.

of 5% for the depreciation rate. Finally, in specifying the relationship between real wage growth and the employment rate, we follow Desai *et al.* (2006), and chose a non-linear specification of $f[.]$: $f[v] = -\epsilon + \frac{\sigma}{(1-v)^{\xi}}$, with $\epsilon = \xi = 1$. As they point out, linear approximations of $f[\cdot]$, often used in the literature for their simplicity, come at the price of the dynamics of employment being likely to leave the unit interval under empirically–based parametric calibrations of the model.[9]

We are left with a set of variables that are not readily found in the data, and therefore need to be calibrated 'internally'; that is, as solution values that, given the other parameters and the endogenous moments to match, put our dynamical system to rest. These variables are (i) the long–run value for capitalist consumption adjusted for productivity, d_{ss}; (ii) the scale parameter in the innovation function a; and (iii) the extent of labor bargaining strength σ. Yet, our dynamical system is made of four equations, and to meet a steady state for all endogenous variables we need to internally calibrate another parameter: we chose the constant output/capital ratio B. While its long–run value can be found in the data and lies in the neighborhood of .4 ($K/Y = 2.5$), our model will be a good match if its steady state returns a value that is in line with such evidence. The internal calibration returns a close fit for B at .426, together with a .0356 value for a, and a .0612 value for the conflict parameter σ. The latter is slightly above the 5% proposed, although without empirical justification, by Desai *et al.* (2006). The corresponding long–run solution value for d is just shy of 12.6%, while the implied growth rate of the economy at the steady state $g_A = g_Y$ is just below 2.1%, also a good match with post–war US data. Table 1 summarizes our calibration.

Parameter	Calibration	Source
ρ	.07	Mehra and Prescott, 1985
s	.044	Impullitti, 2010
δ	.05	standard
χ	.15	Kortum, 1993
a	.0356	calibrated internally
σ	.0612	calibrated internally
B	.426	calibrated internally

Table 1: Baseline parameter calibration.

Figure 1 characterizes the steady state as the intersection between the (Ω) and (N) isoclines in the (ω, n) plane; notice that in the calibrated case the (N) isocline is upward sloping.

3.2 Comparative dynamics

The two steady state conditions (Ω) and (N) highlight the fundamental role that capitalists' intertemporal preferences, labor market conditions, and R&D policy play in determining the equilibrium outcome.

[9]Desai *et al.* (2006) and Harvie *et al.* (2007) both point out that, in general, a savings rule like (3) does not rule out the possibility that the labor share exceeds unity along the transitional dynamics. Yet, the functional specifications they propose are ad hoc, and thus not easily justified as the outcome of dynamic optimization. Given that dynamic trade–offs are the main element at work in our contribution, we made sure that our chosen parametric values also keep the labor share bounded below unity along the transition path.

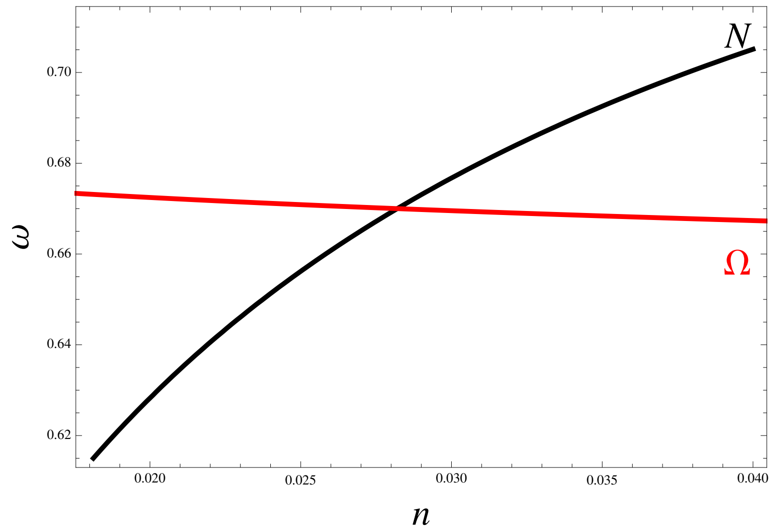

Figure 1: Distribution and R&D isoclines in the baseline case.

3.2.1 Discount rate

The discount rate is the parameter regulating capitalists' preferences between current and future consumption, and therefore their savings propensity. A higher ρ means that asset owning households value future consumption less. We can use equation (R) to analyze the effect of a change in ρ on the steady state equilibrium. Let us start by noting that, given (R), an economically meaningful n requires

$$\frac{h[n_{ss}](1 - B^{-1}(\rho + \delta + \phi[n_{ss}])}{1 - s} > n_{ss}. \tag{8}$$

We show in the Appendix that total differentiation of (R) with respect to n and ρ, while using (8), implies that $h'[.] < 1-s$ is a sufficient condition for $dn/d\rho < 0$. A reduction in the steady state R&D investment share, since $h'[n] > 0$, yields a lower employment rate. Stronger preferences for current consumption reduce overall saving thus lowering both R&D intensity and capital accumulation. Since in the new steady state productivity growth is lower, a constant labor share requires a reduction in wage growth. This is obtained as a consequence of the smaller employment rate v, which demands capital accumulation to slow more than productivity growth. A priori, we cannot define the effect on the equilibrium labor share. Following the increase in the discount rate the (Ω) isocline shifts down to the left, so that the reduction in n will bring about a lower (higher) equilibrium wage share if the (N) isocline is upward (downward) sloping. In our calibration, the (N) isocline proves upward sloping, so that the wage share decreases with an increase in the discount rate: the reduction in the employment rate causes a negative response in wage growth that more than compensates the reduction in productivity gowth.[10]

[10] As formally discussed in the Appendix, the (N) isocline is more likely to slope downward the higher the elasticity of employment to R&D intensity, i.e. when $h'[.]$ is relatively high; since $h[.] \equiv f^{-1}[.]$, a large response in employment means a small reaction in wage growth. In this case, a reduction in productivity growth is accompanied by a smaller decrease in wage growth, which implies a rise in the labor share. On the contrary, when $h'[.]$ is relatively low, a stronger (negative) wage response more than compensates the reduction in productivity and the labor share decreases.

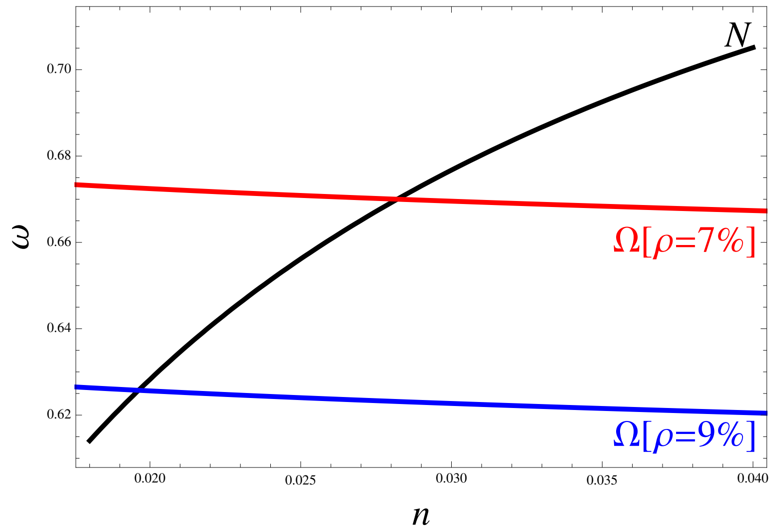

Figure 2: The effect of an increase in the discount rate.

ρ	d_{ss}	n_{ss}	ω_{ss}	v_{ss}	g_Y
.07	.125943	.0282	.67	.94	.0208695
.08	.152812	.0233442	.64794	.939966	.0202862
.09	.178551	.0196192	.625738	.939935	.0197641
.10	.203499	.0166827	.603424	.939907	.0192892
.11	.227877	.0143164	.581023	.939881	.0188516

Table 2: Steady state values corresponding to increasing discount rates. Baseline values are reported in the first row.

Table 2 provides BGP numerical values for the endogenous variables as well as the growth rate, corresponding to increases in the discount rate relative to the baseline case. The distributional effects of lower propensities to save are quite dramatic, as it is visually apparent from Figure 2. For instance, a 2% absolute increase in the discount rate results in an absolute decrease in the labor share of about 4.26%, together with an absolute drop in the R&D intensity of approximately 8.5/10 of a percentage point. The employment effect, on the other hand, is negative but pretty negligible, as it can be seen from the fourth column in Table 2.

3.2.2 Labor bargaining strength

Consider now the influence of labor market conditions on the steady state. Let σ, the measure of workers' bargaining power, be a shift variable in $f[.]$, with $df/d\sigma > 0$; workers can achieve higher wage growth if they manage to increase their bargaining power. Since $h[n_{ss}, \sigma] \equiv f^{-1}[n_{ss}, \sigma]$, we have $dh/d\sigma < 0$. Total differentiation of (R) w.r.t. n and σ, while using (8), shows that $h'[.] < 1 - s$ is a sufficient condition for $dn/d\sigma < 0$. Under such circumstances, a change in σ leaves the (Ω) isocline unaffected while shifting upward the (N) isocline. Accordingly, the reduction in n will bring about an increase in the equilibrium labor share regardless of the slope of (N). The employment rate declines as $h[.]$ is increasing in n and decreasing in σ. Higher wage growth reduces resources available

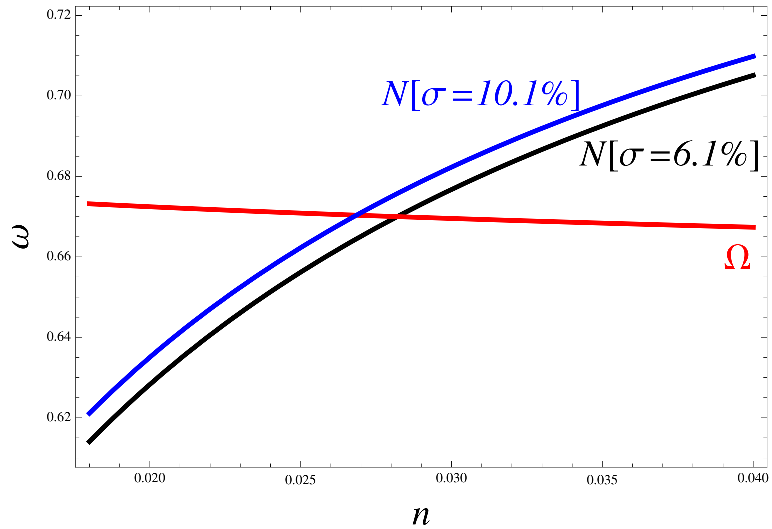

Figure 3: The effect of an increase in workers' bargaining strength.

for investment in R&D and capital accumulation, which both decline. However, to stabilize the wage share, wage growth need be made compatible with the lower productivity growth; this is achieved with a reduction in the employment rate, which tames wage inflation. The decline in the employment rate shows that capitalists respond to higher wage growth by reducing capital accumulation more than they reduce R&D investment. The overall reduction in total investment is accompanied by a relative change in the allocation of invested resources towards improving productivity; the higher labor share makes it more profitable to invest in R&D in order to reduce labor demand.

Table 3 displays the effects of subsequent increases in σ on the steady state in the calibrated version of our model, while Figure 3 plots the shift in the (N) corresponding to an increase of 4% in the degree of conflictuality in the labor market (so that the corresponding steady state values can be found in the first and the last row of Table 3). Due to the fact that the calibrated (Ω) is quite flat, the equilibrium effect is proportionally larger on R&D intensity than on the labor share: a 4% increase in labor market conflict implies a reduction of R&D effort of roughly .135% in terms of the baseline, while it results in a labor share increase of 3.5/10 of a percentage point, as compared to the baseline case. Yet, and due to the shape of the $h[n]$ function, the employment effects of more tense labor relations are more pronounced than the distributional effects, as it can be seen by looking at the last column of Table 3: the decrease in the employment rate corresponding to the plot in Figure 3 is in the realm of 3.9% relative to the baseline value.

3.2.3 Innovation subsidies

Similarly, it is easy to show that $h'[.] < 1 - s$ is a sufficient condition for $dn/ds > 0$. In this case, the reduced cost of investing in R&D yields a higher steady state R&D intensity. Since the (Ω) isocline does not shift in response to a change in the subsidy, equilibrium will move along (Ω) down to the right, so that the labor share decreases. Higher R&D investement raises productivity growth, thus reducing the labor share; the lower wage share makes investing in physical capital more profitable, so that capital accumulation can pick up and provide the increase in the employment ratio necessary

σ	d_{ss}	n_{ss}	ω_{ss}	v_{ss}	g_Y
.0612	.125943	.0282	.67	.94	.0208695
.0712	.124675	.0278614	.670089	.930202	.0208317
.0812	.123406	.0275232	.670178	.920403	.0207936
.0912	.122137	.0271856	.670268	.910603	.0207551
.1112	.120867	.0268485	.670359	.900803	.0207163

Table 3: Steady state values corresponding to increasing bargaining parameters. The first row reports baseline values.

to raise wage growth and stabilize the wage share.

The effects of subsequent increases in the R&D subsidy are reported in Table 4, while Figure 4 plots the shift of the R&D isocline corresponding to a 4% increase in the innovation subsidy. Proportionally, the R&D effort increases by 5.05% relative to the baseline, while the proportional negative effect on the labor share is small, and in the realm of .5/10 of a percentage point. The (positive) variation in employment is negligible.[11]

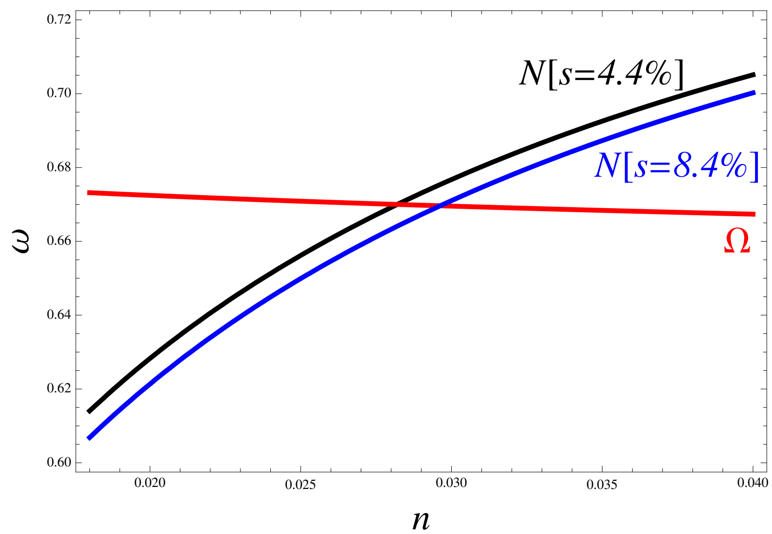

Figure 4: The effect of an increase in the innovation subsidy.

3.3 Discussion

Augmenting the Goodwin growth model with an endogenous innovation intensity determines steady state implications that are fundamentally different from those found within the theory of growth cycles with endogenous direction of technical change. To illustrate the case, let us provide a stripped down version of the dynamical system presented in Shah and Desai (1981). In our notation:[12]

[11]Being the variations relatively small, we distinguish between proportional and absolute percentage changes: for instance, a proportional variation in the labor share is calculated as n_{ss}[s=.084]-n_{ss}[s=.044]/n_{ss}[s=.044], while an absolute variation would be n_{ss}[s=.084]-n_{ss}[s=.044].

[12]Shah and Desai (1981), like Goodwin, assumed zero depreciation and 100% of profits reinvested in accumulation. Thus, their employment equation misses both the discount rate and the depreciation rate relative to (5). Still, the

s	d_{ss}	n_{ss}	ω_{ss}	v_{ss}	g_Y
.044	.125943	.0282	.67	.94	.0208695
.054	.125599	.028544	.669911	.940002	.0209075
.064	.125247	.028896	.669821	.940004	.0209459
.074	.124888	.029256	.66973	.940007	.0209849
.084	.12452	.029625	.669637	.940009	.0210243

Table 4: Steady state values corresponding to increasing R&D subsidy. The first row reports baseline values.

$$\frac{\dot{B}}{B} = -\psi[\omega], \tag{9}$$

$$\frac{\dot{v}}{v} = (1-\omega)B + \frac{\dot{B}}{B} - \frac{\dot{A}}{A} = (1-\omega)B - \psi[\omega] - g[\psi[\omega]], \tag{10}$$

$$\frac{\dot{\omega}}{\omega} = \left(f[v,\sigma] - g[\psi[\omega]]\right), \tag{11}$$

where B, differently from our model, is not constant, and the growth rates of labor productivity and output/capital ratio (respectively $g[.]$ and $\psi[.]$) fall out of the hypothesis of induced innovation (Kennedy, 1964). From (9), we find the steady state share of labor as $\omega_{ss} = \psi^{-1}[0]$, uniquely determined by the properties of the innovation technology. Hence, a shift of the bargaining strength in favor of workers, say an increas in σ, will only affect the employment rate v through (11) while leaving the labor share unaltered. Accordingly, workers' bargaining strength can only influence long–run employment, and not the distribution of income: any attempt to increase wage growth beyond the (exogenous) steady state rate of labor productivity growth will be neutralized by a reduction in employment. In fact, in a framework with induced bias, reducing the level of social conflict over distribution emerges as an unambiguously positive strategy for workers willing to maintain high employment levels.

The situation changes when innovation is labor–augmenting only, but costly as in the present framework. Workers now have the possibility of increasing tension in the labor market in order to bargain for a higher income share, while accepting a reduction in the employment rate: a trade-off between distribution and employment emerges.

3.4 Transitional Dynamics

3.4.1 Stability

The baseline parameterization produces the following eigenvalues for the Jacobian matrix evaluated at the steady state:

$$\begin{aligned} \theta_1 &= -.00359981 + 2.13858i; & \theta_2 &= -.00359981 - 2.13858i; \\ \theta_3 &= .100662; & \theta_4 &= .0360379. \end{aligned}$$

main result we focus on in this section is not dependent on the differences in the behavior of capital accumulation.

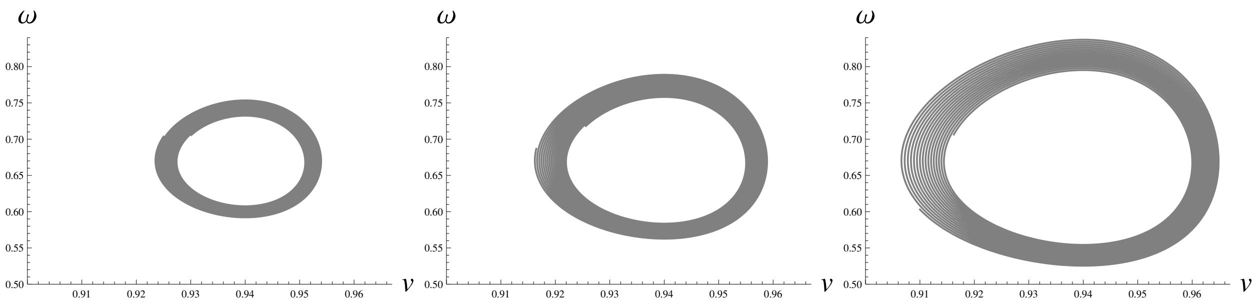

Figure 5: Transitional dynamics: cycles in the employment rate and the labor share for initial conditions that are increasingly farther away from the steady state (left to right).

There are two real positive eigenvalues, and a pair of complex conjugate eigenvalues with negative real parts. The presence of two positive roots rules out full stability of the steady state. However, a steady state is said to be *conditionally stable* if we can pick enough initial conditions in order to annihilate the unstable roots (Gandolfo, 1997, Chapter 18 and 22). The optimal control nature of accumulation and innovation provides two initial conditions on the control variables c and R that can be chosen given the initial values on capital stock K and the state of technology A at the beginning of the planning horizon. These two initial conditions can be picked freely in order to let the system jump onto its stable manifold, the dimension of which is equal to the number of eigenvalues with negative real parts: two, in this case. Because the stable subsystem contains imaginary roots, the convergence to the equilibrium will occur in cyclical fashion. Thus, our model with perfect foresight and the choice of resources to allocate to innovative activity leads to dynamics that are similar to those of Shah and Desai (1981); Foley (2003); Julius (2005): there are counterclockwise, dampened oscillations in the (v, ω) plane. Yet, there are at least two important differences in the character of the economy along the transition to the steady state. First, in the induced technical change model, innovation is purely labor–augmenting *only at the steady state,* so that the dampened cycles are necessary in order to rid of capital–augmenting technological progress. Conversely, here innovation is purely labor–augmenting over the entire transition path, but what varies over the cycle is the composition of capitalist spending between accumulation and innovation. Second, differently from the induced technical change setup, in the present framework there is no immediate direct feedback from the labor share to the growth rate of labor–augmenting technology. Instead, the interaction between labor productivity growth and income distribution cannot be determined without taking into account the dynamics of employment and productivity–adjusted consumption. Figure 5 displays trajectories for the labor share plotted against the employment rate for three different initial conditions. For these plots, the terminal time is set to 50 periods.

3.4.2 Sensitivity Analysis

The discount rate ρ and the elasticity of the innovation technology χ are the two externally calibrated parameters for which there is a considerable degree of uncertainty in the literature. Therefore, it makes sense to assess whether changes in either parameter produce substantial changes in the dynamics of the model. We know that the steady state values of our endogenous variables will change in response to parametric changes; so will the eigenvalues of the Jacobian matrix. In particular, what is of interest is whether the real parts of the complex conjugate eigenvalues $\theta_{1,2}$ become zero,

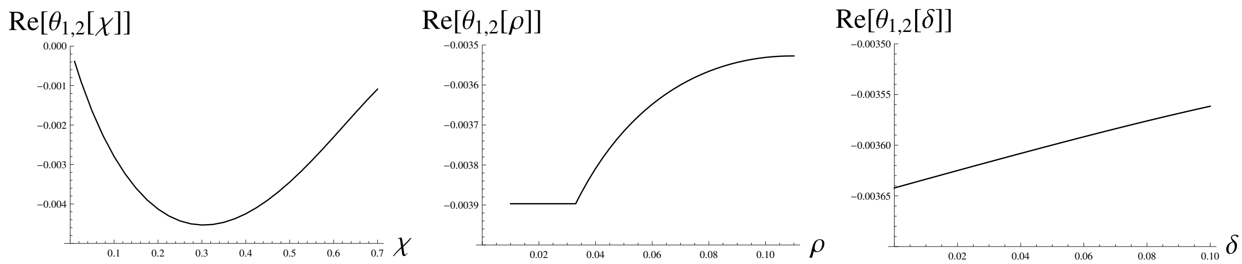

Figure 6: Sensitivity analysis.

or even change in sign, for certain parameter values.[13] In the former case, the model would undergo a Hopf bifurcation and reproduce the closed orbits found in the original Goodwin (1967) model. In the latter case, instead, the steady state would turn to a source, and the cycles would move away from it. To check whether such occurrances were possible, we let the innovation elasticity and the discount rate vary separately in small steps; then, we recalculated the steady states and the eigenvalues of the Jacobian matrix at each step, and plotted the corresponding real parts of the complex conjugate eigenvalues $\theta_{1,2}$ against the parameter set to vary. In the plots in Figure 6, χ is set to vary from .01 to .7, which includes the [.1, .6] interval of point estimates reported by Kortum (1993) as well as a 10% error band for those estimates (given that Kortum, 1993 does not report confidence intervals), while the discount rate ρ is let vary from .01 to .11. For completeness, we also let the depreciation rate vary from 0 to 10%.

As it can be seen, the real parts of the stable eigenvalues are flat for low values of the discount rate, and then monotonically increasing, but never change in sign in response to increases in ρ. Similarly, the complex conjugate eigenvalues keep having negative real parts within the specified range for the innovation elasticity. The same is true regarding depreciation. Thus, our findings regarding conditional stability and dampened oscillations appear to be robust to parametric uncertainty on the externally calibrated parameters of the model.

4 Conclusions

This paper presented a long–run growth and distribution framework combining elements of mainstream endogenous growth theory with the class structure and distributional conflict at the heart of the Goodwin (1967) model. The economy is described by a four-dimensional dynamical system tracing the evolution of capitalist consumption adjusted for productivity, the intensity of R&D efforts by firms, the labor share, and the employment rate. The steady state of the model is conditionally stable, and the corresponding path is characterized by dampened oscillations where the extent of distributive conflict gradually fades out. Being this one a model —like Goodwin's— in which Say's law holds at any moment in time, we find that lower preference for current relative to future consumption (that is, a higher propensity to save out of profits) will foster both accumulation and innovation, eventually determining higher labor productivity growth, higher employment, and a higher labor share at the steady state. Further, we showed that innovation subsidies also feed into the distributional conflict as they determine an increase in the long run growth rate of labor productivity which, in the long run, produces a reduction in the labor share. Finally, and differ-

[13]We also evaluated the unstable eigenvalues, and found no change in sign within our parametric ranges.

ently from previous related contributions on the growth cycle with endogenous technical change, we showed that a stronger bargaining position for workers allows them to increase the long run share of labor at the expenses of employment. With respect to both the steady state and the transitional dynamics, we emphasized the different implications of endogenizing the intensity as opposed to the direction of technical change. To this extent, even though we presented simulations by calibrating the model with empirically based parameters, our contribution is not meant to enter the debate over the empirical plausibility and implications of the Goodwin growth cycle (a recent account of which is provided in Fiorio, Mohun and Veneziani, 2013), especially in light of the two competing approaches to technological progress. Empirical tests aimed at addressing the contribution of the intensity, as opposed to the direction, of technical change, seem a fruitful area for further research.

References

Aghion, P., and Howitt, P., 2010. *The Economics of Growth* Cambridge, MA: MIT Press.

Blanchard, O., and Kahn, C., 1980. The Solution of Linear Difference Models under Rational Expectations. *Econometrica*, 48: 1303-11.

Desai, M., Henry, B., Mosley, A., and Penderton, M., 2006. A Clarification of the Goodwin Model of the Growth Cycle. *Journal of Economic Dynamics and Control* 30: 2661-2670.

Fiorio, C., Mohun, S., and Veneziani, R., 2013. Social Democracy and Distributive Conflict in the UK, 1950-2010. Working Paper No. 705, School of Economics and Finance, Queen Mary University of London. ISSN: 1473-0278.

Foley, D.K., and Michl, T., 1999. *Growth and Distribution*. Cambridge, MA: Harvard University Press.

Foley, D. K., 2003. Endogenous Technical Change with Externalities in a Classical Growth Model. *Journal of Economic Behavior and Organization*, 52: 167-189.

Gandolfo, G., 1997. *Economic Dynamics* Springer–Verlaag.

Goodwin, R., 1967. A Growth Cycle. In: Carl Feinstein, editor, *Socialism, capitalism, and economic growth*. Cambridge, UK: Cambridge University Press.

Harvie, D., Kelmanson, M.A., Knapp, D.G., 2007. A Dynamical Model of Business–Cycle Asymmetries: Extending Goodwin. *Economic Issues*, 12(1): 53–92.

Impullitti, G., 2010. International Competition and U.S. R&D Subsidies: a Quantitative Welfare Analysis. *International Economic Review*, Vol. 51, issue 4, November 2010.

Julius, A. J., 2005. Steady-state growth and distribution with an endogenous direction of technical change. *Metroeconomica*, 56 (1): 101-125.

Kennedy, C., 1964. Induced Bias in Innovation and the Theory of Distribution. *Economic Journal*,

Kortum, S., 1993. Equilibrium R&D Ratio and the Patent-R&D Ratio: U.S. Evidence. *American Economic Review*, Papers and Proceedings, 83: 450-457.

Mehra, R., and E.C. Prescott, 1985. The Equity Premium: a Puzzle. *Journal of Monetary Economics* 15, 145-161.

Nordhaus, W. 1971. Some Skeptical Thoughts on the Theory of Induced Innovation. *Quarterly Journal of Economics*,

Shah, A., and Desai, M. 1981. Growth Cycles with Induced Technical Change. *Economic Journal*, 91: 1006-10.

Smulders, S. and van de Klundert, T., 1995. Imperfect Competion, Concentration and Growth with Firm-specific R&D. *European Economic Review*, 39: 139-60.

van der Ploeg, F., 1987. Growth Cycles, Induced Technical Change, and Perpetual Conflict over the Distribution of Income. *Journal of Macroeconomics*, 9: 1-12.

A Dynamic Optimization

The current–value Hamiltonian is:

$$\mathcal{H} = \ln c + \mu \left(B \left(1 - \frac{w}{A} \right) K - \delta K - c(1+\tau) - R(1-s) \right) + \lambda \phi[n] A,$$

where λ is the current–value costate variable. The first order conditions are:

$$c^{-1} = \mu(1+\tau) \tag{12}$$

$$\mu(1-s) = \lambda\phi'[n] \tag{13}$$

$$\rho\mu - \dot{\mu} = \mu B(1-\omega) \tag{14}$$

$$\begin{aligned} \rho\lambda - \dot{\lambda} &= \mu BKw \left(\tfrac{1}{A^2} \right) + \lambda(\phi[n] - \phi'[n]n) \\ &= \mu\omega L + \lambda(\phi[n] - \phi'[n]n) \end{aligned} \tag{15}$$

plus two transversality conditions. From (14), we get $\rho - \dot{\mu}/\mu = B(1-\omega) - \delta$, which, given (12), yields $\dot{c}/c = B(1-\omega) - (\rho+\delta)$. Next, differentiate log of (13) to get

$$\frac{\dot{\lambda}}{\lambda} - \frac{\dot{\mu}}{\mu} = (1-\chi)\frac{\dot{n}}{n}$$

and use (13), (14) and (15) to find, first:

$$\rho - \frac{\dot{\lambda}}{\lambda} = \left(\phi[n] + \phi'[n] \left(\frac{\omega v}{1-s} - n \right) \right),$$

and then $\dfrac{\dot{\lambda}}{\lambda} - \dfrac{\dot{\mu}}{\mu} = \rho - \frac{\dot{\mu}}{\mu} - \left(\rho - \frac{\dot{\lambda}}{\lambda} \right) = B(1-\omega) - \delta - \left(\phi[n] - \phi'[n] \left(n - \frac{\omega v}{1-s} \right) \right)$, from which equation (4) follows.

B Comparative Dynamics

B.1 Discount rate

Start with (R), and totally differentiate it w.r.t. n and ρ to find:

$$\begin{aligned} d\rho \quad (1+\tfrac{h[n_{ss}]\phi'[n_{ss}]}{(1-s)B}) = \quad & \phi''[n_{ss}]\left(\tfrac{h[n_{ss}]}{1-s}\left(1-\tfrac{\rho+\delta+\phi[n_{ss}])}{B}\right)-n_{ss}\right)dn \\ + \qquad\qquad & \phi'[n_{ss}]\left(\tfrac{h'[n_{ss}]}{1-s}\left(1-\tfrac{\rho+\delta+\phi[n_{ss}])}{B}\right)-\tfrac{h[n_{ss}]\phi'[n_{ss}]}{(1-s)B}-1\right)dn. \end{aligned}$$

Given $\phi''[n_{ss}]<0$ and (8), $h'[n_{ss}]\left(1-\frac{\rho+\delta+\phi[n_{ss}])}{B}\right)/(1-s)-h[n_{ss}]\phi'[n_{ss}]/((1-s)B)-1<0$ implies $dn/d\rho<0$. Using the definition of $\phi[n_{ss}]$ and rearranging the previous condition yields

$$\frac{h'[n_{ss}]}{1-s}<\frac{h[n_{ss}]\phi'[n_{ss}]}{(1-s)B}\left(1-\frac{\rho+\delta+\phi[n_{ss}])}{B}\right)^{-1}+\left(1-\frac{\rho+\delta+\phi[n_{ss}])}{B}\right)^{-1}. \tag{16}$$

From (8), $h[n_{ss}]/n(1-s)>\left(1-\frac{\rho+\delta+\phi[n_{ss}])}{B}\right)^{-1}$,

therefore $\frac{h[n_{ss}]\phi'[n_{ss}]}{(1-s)B}\left(1-\frac{\rho+\delta+\phi[n_{ss}])}{B}\right)^{-1}+\left(1-\frac{\rho+\delta+\phi[n_{ss}])}{B}\right)^{-1}=\left(1-\frac{\rho+\delta+\phi[n_{ss}])}{B}\right)^{-1}(1+\zeta)>1$, where ζ is an unknown positive scalar. The right hand side of (16) is strictly larger than one so that $h'[n_{ss}]<1-s$ is a sufficient condition for $dn/d\rho<0$.

B.2 Labor market conditions

Rewrite (R) as

$$\rho=\phi'[n_{ss}]\frac{h[n_{ss},\sigma](1-B^{-1}(\rho+\delta+\phi[n_{ss}]))}{1-s}-\phi'[n_{ss}]n_{ss}$$

to make the role of bargaining power explicit. Totally differentiate it w.r.t. n and σ to find:

$$\begin{aligned} d\sigma\left(-\frac{dh}{d\sigma}\tfrac{\phi'[n_{ss}])}{1-s}\left(1-\tfrac{\rho+\delta+\phi[n_{ss}])}{B}\right)\right) \quad = \quad & \left[\phi''[n_{ss}]\left(\frac{h[n_{ss},\sigma]}{1-s}\left(1-\frac{\rho+\delta+\phi[n_{ss}])}{B}\right)-n_{ss}\right)\right. \\ + \quad & \left.\phi'[n_{ss}]\left(\frac{dh}{dn}\frac{\left(1-\frac{\rho+\delta+\phi[n_{ss}])}{B}\right)}{1-s}-\frac{h[n_{ss},\sigma]\phi'[n_{ss}]}{(1-s)B}-1\right)\right]dn. \end{aligned}$$

After noting that $-\dfrac{dh}{d\sigma}\left(\frac{\phi'[n_{ss}](1-\rho-\phi[n_{ss}])}{1-s}\right)>0$, proceed as in B.1 to find that $h'[n_{ss}]<1-s$ is a sufficient condition for $dn/d\sigma<0$.

B.3 R&D subsidy

Totally differentiate (R) w.r.t. n and s to find:

$$\begin{aligned} -\left(\phi'[n_{ss}]\tfrac{h[n_{ss}]}{(1-s)^2}\left(1-\tfrac{\rho+\delta+\phi[n_{ss}])}{B}\right)\right)ds \quad = \quad & \left[\phi''[n_{ss}]\left(\tfrac{h[n_{ss}]}{1-s}\left(1-\tfrac{\rho+\delta+\phi[n_{ss}])}{B}\right)-n_{ss}\right)\right. \\ + \quad & \left.\phi'[n_{ss}]\left(\tfrac{h'[n_{ss}]}{1-s}\left(1-\tfrac{\rho+\delta+\phi[n_{ss}])}{B}\right)-\tfrac{h[n_{ss}]\phi'[n_{ss}]}{(1-s)B}-1\right)\right]dn. \end{aligned}$$

Since $\phi'[n_{ss}]\frac{h[n_{ss}](1-\rho-(\gamma-1)\phi[n_{ss}])}{(1-s)^2}>0$ proceeding similarly to F.1 and F.2 yields $h'[n_{ss}]<1-s$ as a sufficient condition for $dn/ds>0$.

B.4 On the slope of the N isocline

Our calibrated economy already proves the possibility that the (N) isocline be upward sloping. Here we provide a sufficient condition for the isocline to be downward sloping. Keeping in mind that $\phi[n] = an^{\chi}$, differentiate (N) w.r.t. ω and n to find:

$$-d\omega\left(B + \frac{1}{1-s}\phi^{'}[n_{ss}]h[n]\right) = dn\left[\omega\left(\frac{h'[n]}{(1-s)}\phi^{'}[n_{ss}] + \frac{h[n]}{(1-s)}\phi^{''}[n_{ss}]\right) + (1-\chi)\phi^{'}[n_{ss}]\right].$$

Divide the both sides of the previous equation by $\phi^{'}[n_{ss}]$ to find

$$-d\omega\left(\frac{B}{\phi'[n_{ss}]} + \frac{1}{1-s}h[n]\right) = dn\left[\omega\left(\frac{h'[n]}{(1-s)} - (1-\chi)\frac{h[n]}{(1-s)n}\right) + (1-\chi)\right].$$

Accordingly, $h'[n] > (1-\chi)\dfrac{h[n]}{n} \Rightarrow \dfrac{d\omega}{dn} < 0$. In other words, a sufficient condition for a negative slope of the (N) isocline is that the elasticity of the employment rate to R&D share is larger than $1-\chi$.

Publisher: Hans-Böckler-Stiftung, Hans-Böckler-Str. 39, 40476 Düsseldorf, Germany
Phone: +49-211-7778-331, IMK@boeckler.de, http://www.imk-boeckler.de

IMK Working Paper is an online publication series available at:
http://www.boeckler.de/imk_5016.htm

ISSN: 1861-2199

Hans Böckler Stiftung

Fakten für eine faire Arbeitswelt.